LABORATORIUM VOOR SCHEEPSCONSTRUCTIES

TECHNISCHE HOGESCHOOL - DELFT

RAPPORT Nr. SSL 101

BETREFFENDE:

Proefschrift van R. Nielsen jr.

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1. Mogelijkheden en beperkingen van de berekeningsmethode

De methode die de auteur ontwikkeld heeft maakt het mogelijk om grote, orthogonaal verstijfde plaatconstructies op doorbuiging en krachtopname te berekenen. Hulp van een electronische rekenmachine en een groot aantal sub-programma's in procedurevorm is noodzakelijk, maar dan is ook zeer veel mogelijk. Dit kan het beste blijken uit een bespreking van wat voor andere methoden vaak beperkingen zijn.

a) Belastingen. Hieraan worden geen beperkingen gesteld zolang zij te herleiden zijn tot belastingen op de stijlen. Puntlasten op een plaatpaneel zijn dus niet zonder meer te behandelen, maar behoren ook niet voor te komen.

b) Randvoorwaarden. Moeten bekend zijn.

c) Profielafmetingen. Dienen constant te zijn over de lengte van het betreffende constructiedeel.

d) Stijlafstanden. Inconstante stijlafstand wordt door de auteur niet behandeld. (pag. 27, § 1).

e) Torsiestijfheden. Kunnen in rekening gebracht worden.

De methode heeft volgens Nielsen weinig bezwaren behalve dat het aantal stijlen voldoende groot moet zijn. Beperkingen worden gesteld doordat de formules hanteerbaar moeten blijven en doordat het geheugen van de computer niet onbegrensd is.
Verder blijft het een methode waarbij een orthogonaal verstijfd plaatveld als open raster wordt beschouwd. (zie 2). Het is dus altijd een benaderingsmethode.

Bij de gewone rasterberekening wordt de statisch onbepaalde constructie, statisch bepaald gemaakt door alle staven tussen de knooppunten los te nemen. De continuïteitsvoorwaarden voor elk knooppunt vormen de basis voor een systeem van vergelijkingen waarmee de onbekenden (reacties of verplaatsingen) worden opgelost. Nielsen brengt elke serie knooppunten van één drager onder in één differentiaal vergelijking die opgelost moet worden.

2. Grondprincipe.

De rekenmethode die in Nielsen's dissertatie besproken wordt is gebaseerd op het geval van een belaste balk op een verende onderlaag. Dit is bij vrijwel alle roosterberekeningen zo, en de gedachtengang die tot dit uitgangspunt geleid heeft wordt bekend verondersteld.
Een bestudering van een voorgaand artikel van Nielsen en Michelsen (1), dat tevens als inleiding tot de dissertatie kan dienen, werkt verhelderend, zoals blijkt uit het volgende.

Uit de bijgaande copieën van de eerste twee pagina's van het artikel (1) zijn de aangestreepte fragmenten van belang. Zij worden hieronder in de tekst opgenomen, de copieën zijn volledigheidshalve bijgevoegd. De alinea boven fig. 2 luidt: "In many engineering problems such as grillage ship structures, one encounters beams not supported partially by a continuous elastic medium but rather by a finite number of equally spaced springs. A simple case is the deck girder supported partially by the deck beams."
Dit mag dan voor een rasterwerk-analyst normaal zijn, voor een praktische scheepsbouwer is dat niet het geval. De laatste beschouwt een dekdrager als steunpunt voor de balken en niet omgekeerd. Verderop (pag. 2) wordt alles duidelijk. Het begin ligt bij de aangestreepte alinea onder fig. 3 en de sleutel wordt gevormd door de formules (0.4) en (1.4), die volgens simpele statica afgeleid zijn. Het geheel luidt als volgt:
"It is therefore required to obtain a relationship between the deflection of any stiffener and the force exerted by the stiffener onto the girder. To be able to account for the live load q, a uniform load of qa per unit length must be considered carried by the stiffeners. The freebody diagram of a stiffener, including effective plating, is shown in Figure 4, where R represents the force between girder and stiffener. The deflection at R due to the uniform load is given by

$$y_r(q) = \frac{5\, q\, a\, b^4}{384\, EI_s}$$

whereas the deflection due to the reaction R at the point of application is given by

$$y_R\,(R) = -\frac{Rb^3}{48\, EI_s}$$

The total deflection is therefore

$$y_R = y_R(q) + y_R(R) = \frac{5\,q\,a\,b^4}{384\,EI_s} - \frac{R\,b^3}{48\,EI_s} \qquad (1.3)$$

Since R represents the loading on the girder, this is the quantity sought, and, from Equation (1.3), it follows that

$$R = \frac{5}{8}\,qab - \frac{48\,EI_s}{b^2}\,y_R$$

If this concentrated force is assumed to be uniformly distributed over a stiffener spacing, one can say that the girder is loaded by a distributed load of magnitude

$$\frac{R}{a} = \frac{5}{8}\,qb - \frac{48\,EI_s}{ab^3}\,y \qquad (1.4)$$

where the subscript has been dropped since Equation (1.4) is valid over the whole length of the girder. In addition to distributing the elastic reaction $\frac{48\,EI_s}{b^3}\,y$, it is noted that Equation (1.4) also distributes the concentrated load uniformly over a stiffener spacing. This fact is not too serious as far as the elastic reaction is concerned if the number of stiffeners is sufficiently large, and in regard to the other part of the loading on the girder, it will be shown that no advantage is to be gained by distributing this since concentrated forces can be handled quite readily."

Uit formule (1.4) is nu evident waarom de drager beschouwd wordt als een belaste en verend-ondersteunde ligger.

In de dissertatie wordt de vergelijking voor de elastische lijn van de verend ondersteunde ligger op een andere manier afgeleid dan in (1) i.v.m. het streven naar een zeer algemene behandeling van het probleem met als consequentie een streng mathematische algemene formulering van de randvoorwaarden. De differentiaalvergelijking of de gekoppelde differentiaal vergelijkingen wordt of worden opgelost met behulp van Laplace transformaties. Dit is een wiskundige methode waarbij een differentiaal vergelijking wordt getransformeerd tot een lineaire algebraïsche vergelijking. Een definitie is te vinden in (2) (hoofdstuk XIII).

3. Algemeen

De verdiensten van het werk worden het beste geïllustreerd door eens na te gaan wat er van het lijstje bezwaren overblijft dat Ando (3) geeft in een overzichtje van plaatveldberekeningsmethoden. (Zie bijgaande copieën).

Bezwaar I

De torsiestijfheid van de stijlen kan in rekening worden gebracht. (Heeft meestal weinig invloed (Nielsen pag. 55 e.v.))

De torsiestijfheid van de stringers kan in rekening worden gebracht middels een iteratieproces dat snel zal convergeren. De torsiestijfheid van dubbele bodems mag niet verwaarloosd worden.

Bezwaren II t/m IV Vervallen.

Literatuur

(1) Michelsen, F.C. and Nielsen, R. "Analysis of Grillage Structures by means of the Laplace Transform."
Schiffstechniek Bd. 9 - 1962 - Heft 49.

(2) Handboek der Wiskunde red. Kuipers, L en Timman, R.

(3) Ando, N. "On the Strength of orthogonally stiffened plate"
Transp. Techn. Research Inst. Rep. nr. 48, maart 1962.

September 1965.

[signature]

Analysis of Grillage Structures by means of the Laplace Transform

F. C. Michelsen and R. Nielsen, jr.

An analysis of grillage structures is presented which is based on the solution of girder deflections as obtained from Laplace Transforms of governing differential equations. The resulting formulation is applicable to any type of loading, including concentrated forces, and lends itself readily for computer programming. The method of solution therefore offers the opportunity to determine optimum distribution of material with a minimum amount of labor and in a short time.

The analysis of grillage structures has received a great deal of attention in the past. In view of their great importance, this is not surprising. Considering the ship structure as an example, one observes immediately that many of the primary strength members consist of plating supported by mutually orthogonal sets of stiffeners — i. e., a grillage structure.

It is immediately realized that such a structure is highly redundant and that strength calculations would be cumbersome if they were based on elementary beam formulae. Early researchers, among them Timoshenko and Föppl, soon realized that the grillage structure could be analyzed by the use of deflection equations derived for beams on elastic foundations in much the same manner as in the analysis of railroad tracks [5]. Dr. G. Vedeler [2] and Hetenyi [6] have contributed much to perfect this method of analysis, but it can be said that, since their treatments are based on the classical solutions of the governing differential equations, their method becomes quite complicated to handle for some types of loading such as the case of a considerable number of concentrated forces.

The purpose of this paper is to present an analysis of the grillage structure based on the elastic foundation approach. However, the differential equations of deflections are solved by means of Laplace Transforms instead of by the use of classical solutions. This approach leads to a general formulation valid for all types of loading. Subscript notation has been used extensively to save space and to attempt to make the problem more tractable. In doing so, the computer programming was kept in mind, but even more important is the ability of this form of notation to reveal important functional relationship between physical parameters.

1. Beam on an elastic Foundation

A beam supported along its entire length by a continuous linear elastic foundation will have a distributed load which at every point is proportional to the deflection at that point. The load intensity due to the elastic foundation is then represented by Ky, where K is a constant called the modulus of foundation. If the live load on the beam is defined by $f(x)$, the total lateral load on the beam becomes $f(x) - Ky$. Figure 1 shows the loading in the case of a simply supported beam on an elastic foundation.

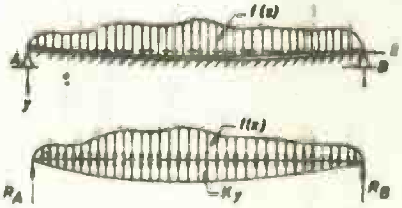

Fig. 1 Simply supported beam on an elastic foundation

For the coordinate system chosen, and with the usual definition of positive bending moment, the following relationships hold:

$$\frac{d^2 y}{d x^2} = -\frac{M}{EI}$$
$$\frac{d^2 M}{d x^2} = -p(x) \qquad (1.1)$$

where $p(x)$ is the total lateral load as a function of x.

From equation (1.1), the differential equation governing the beam on an elastic foundation then becomes

$$EI \frac{d^4 y}{d x^4} + K y = f(x) \qquad (1.2)$$

This equation is, of course, only valid within the assumptions of the Bernoulli-Euler beam theory.

In many engineering problems such as grillage ship structures, one encounters beams not supported partially by a continuous elastic medium but rather by a finite number of equally spaced springs. A simple case is the deck girder supported partially by the deck beams. Schematically, this situation is shown in Figure 2.

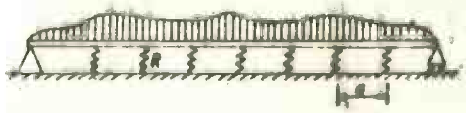

Fig. 2

Such a problem would indeed be difficult to solve if the exact physical nature of the foundation were to be taken into account. If, however, there are a sufficient number of closely spaced springs, the beam can be assumed to be partially supported by an equivalent continuous elastic foundation of modulus $K = \frac{\bar{K}}{a}$, an assumption which is very good for small deflections of beam theory. To solve Equation (1.2), the determination of the equivalent modulus of the foundation is required. Fortunately, this can be done by means of simple beam formulae. As an example, consider the case of Figure 3. A rectangular panel of dimensions L and b is stiffened by one girder in one direction and several closely spaced stiffeners in the other. Without loss of generality, the panel is assumed to be loaded by a uniform load of q lbs. per square inch, and

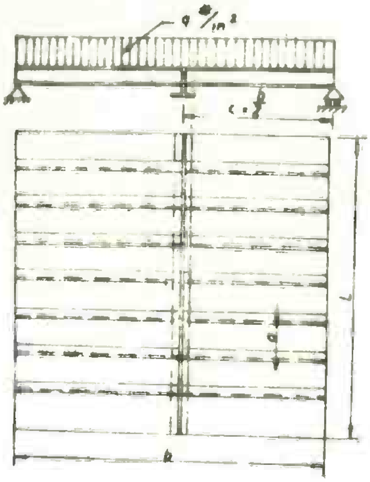

Fig. 3

the stiffeners are assumed to be simply supported at both ends. As mentioned previously, the girder is supported elastically by the stiffeners. It is therefore required to obtain a relationship between the deflection of any stiffener and the force exerted by the stiffener onto the girder. To be able to account for the live load q, a uniform load of qa per unit length must be considered carried by the stiffeners. The freebody diagram of a stiffener, including effective plating, is shown in Figure 4, where R represents the force between girder and stiffener. The deflection at R due to the uniform load is given by

$$y_R(q) = \frac{5\,q\,a\,b^4}{384\,EI_s}$$

whereas the deflection due to the reaction R at the point of application is given by

Fig. 4

$$y_R(R) = -\frac{R\,b^3}{48\,EI_s}$$

The total deflection is therefore

$$y_R = y_R(q) + y_R(R) = \frac{5\,q\,a\,b^4}{384\,EI_s} - \frac{R\,b^3}{48\,EI_s} \qquad (1.3)$$

Since R represents the loading on the girder, this is the quantity sought, and, from Equation (1.3), it follows that

$$R = \frac{5}{8}\,q\,a\,b - \frac{48\,EI_s}{b^3}\,y_R$$

If this concentrated force is assumed to be uniformly distributed over a stiffener spacing, one can say that the girder is loaded by a distributed load of magnitude

$$\frac{R}{a} = \frac{5}{8}\,q\,b - \frac{48\,EI_s}{a\,b^3}\,y \qquad (1.4)$$

where the subscript has been dropped since Equation (1.4) is valid over the whole length of the girder. In addition to distributing the elastic reaction $\frac{48\,EI_s}{b^3}\,y$, it is noted that Equation (1.4) also distributes the concentrated load uniformly over a stiffener spacing. This fact is not too serious as far as the elastic reaction is concerned, if the number of stiffeners is sufficiently large, and, in regard to the other part of the loading on the girder, it will be shown that no advantage is to be gained by distributing this since concentrated forces can be handled quite readily.

The differential equation for the girder for the simple case considered then becomes, by Equation (1.2),

$$E\,I_G\,\frac{d^4 y}{d\,x^4} + \frac{48\,EI_s}{a\,b^3}\,y = \sum_{n=1}^{N} R_n \qquad (1.5)$$

where it is noted that $\frac{48\,EI_s}{a\,b^3}$ represents the spring constant K, and $f(x)$ of Equation (1.2) is equal to $\sum R_n$ which represents a series of concentrated forces, each of magnitude $\frac{5}{8}\,q\,a\,b$ acting at intersections of girders and stiffeners. The solution of Equation (1.2) by means of Laplace Transform techniques for various loadings and geometry is the subject of Reference 1. [Example 1 of Appendix III] shows that Equation (1.2) is a special case of the more general problem of coupled equations treated here.

If the panel is supported by more than one girder, it is advantageous to generalize Equation (1.3) as follows:

$$y_i = d_i - R_j\,\alpha_{ij} \quad (i = j = 1, 2, \cdots N) \qquad (1.6)$$

where y_i = Total deflection of stiffener at the ith girder

d_i = Deflection at the ith girder due to live load and no girder support.

α_{ij} = Influence index — i. e., the deflection of the stiffener at the ith girder due to a unit load at the jth girder.

R_j = Force between jth girder and the stiffener.

N = Number of girders.

The usual convention of summing over repeated subscript is observed. Solving Equation (1.6) for the equivalent distributed loading on the ith girder, one obtains

$$\bar{R}_i = \bar{\alpha}\,(d_i - y_i - R_j\,\bar{\alpha}_{ij}) \qquad (1.7)$$

where

$$\bar{R}_i = \frac{R_i}{a}$$

$$\bar{\alpha} = \frac{1}{a\,\alpha_{ii}} \text{ (no summation)}$$

$$\bar{\alpha}_{ij} = \alpha_{ij}; \quad i \neq j$$
$$= 0; \quad i = j$$

The differential equation for the ith girder then becomes

$$E\,I_{ij}\,\delta_{jk}\,\frac{d^4 y_k}{d\,x^4} = \bar{\alpha}\,(d_i - y_i - R_j\,\bar{\alpha}_{ij}) \qquad (1.8)$$

where

$$\delta_{jk} = 1; \quad j = k$$
$$= 0; \quad j \neq k$$

I_{ij} = Moment of inertia of ith girder if $j = i$
$= 0; \quad j \neq i.$

Ando, N. "On the strength of orthogonally stiffened plate" rep. 48 of Transportation Technical Research Inst. maart 1962.

i. e.,

(1) Method of calculation by regarding it as grid structure.[1)~8)]

(2) Energy method.[10)]

(3) Method of calculation by applying the orthotropic plate theory.

Merits and demerits contained in these methods may be considered as follows.

(1) Grid structure

Merits :

(i) It is very plain to lead the basic equation, and even if the end conditions are these of built in or more complicated, it can do as simple as the case of simply supported edges.

(ii) As the value of local element can be exactly calculated, it may be available for designing details of respective element.

Demerits :

(i) In this method, torsional rigidity is ignored, because every stiffener with effective breadth of plate is cut off and calculated independently as a simple beam. The calculated value, therefore, will differ from actual structure, especially in case of the structure with comparatively large torsional rigidity such as the double bottom of ship.

(ii) Though it is usually very easy to lead basic equations, solving these equations will be considerably troublesome.

(iii) As those equations have different form according to the number of stiffener, loading, end condition, etc., trial calulation will be difficult at the time of designing these structure.

(iv) The way of distributing the given load to respective stiffener is not obtained unless adequate assumptions are given.

(2) Energy method

Merits :

(i) As the solution may be obtained without difficulty by comparatively simple calculation, this is the convenient method for estimating the scantling of these structures, and strength corresponding to variation of scantling will be readily obtained.

Demerits :

(i) Solution obtained will satisfy only one of the basic equation or boundary condition. Consequently, it will contain considerable error.

(ii) There is no adequate method to check the magnitude of error.

2) Orthotropic plate

rits :

(i) As it holds the continuity of plate, and torsional rigidity is considered reasonably, it will be the most precise one among aforesaid methods.

(ii) Values corresponding to given end conditions or loadings can be readily obtained by the formulas or graphs which may be previously prepared.

(iii) As variation of stress or deflection corresponding to the variation of scantlings may be easily observed from the formula derived from orthotropic plate theory, it is very convenient for practical design.

nerits :

(i) It is considerably troublesome to obtain the solution of orthotropic plate with complicated boundary conditions. And the investigation of these solution is still imperfect and this method may not be put to practical use, except very simple cases.

(ii) Correlation between orthogonally stiffened plate and orthotropic plate must be studied for a separated item.

(iii) As the value derived from the orthorotropic plate will give the mean value of orthogonally stiffened plate, the value of local element may not be obtained by itself.

As above mentioned, though every method contains both merits and nerits, demerits of grid structure method is essential, on the other hand, demerits contained in the method by orthotropic plate theory may be ioved by the future research.

From this point of view, it seems to be very important and useful to estigate the solution of orthotropic plate with various loadings and edge ditions to remove the difficulties in case of applying this theory to the hogonally stiffened plate and to establish the method of analysis by this ory.

.3 An outline of investigations about orthotropic plate

Two main branches may be considered about the investigations on orthopic plate theory. One is the investigations of elastic rigidity constants