FAILURE CRITERION FOR PLAIN CONCRETE UNDER SHORT-TIME LOAD

by

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ABSTRACT

Considering plain concrete as an isotropic, homogeneous continuum, a criterion has been proposed to govern the failure of this material. According to this strain-based criterion, two modes of failure are possible, i.e., the material element may fail either in shear or in tension depending upon the corresponding strain (stress) state.

It is shown in this paper that the proposed failure criterion is capable of predicting with great accuracy the experimental results obtained from two different types of test. Moreover, the theory may be applied to predict the mode of failure if the stress states of the samples are carefully controlled. A method has also been proposed for obtaining constant stress and strain states during unconfined compression tests.

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FAILURE CRITERION FOR PLAIN CONCRETE 
UNDER SHORT-TIME LOAD

I. INTRODUCTION

Although concrete is locally nonhomogeneous, multiphased and full of micro-cracks*, this material may macroscopically be considered isotropic and homogeneous, as the orientation of the aggregates is random. For the purpose of engineering design, it is appropriate to consider the concrete as a continuum. The need for a criterion of failure for plain concrete under all states of stress is well-known. Methods for obtaining the load-carrying capacity of concrete structures may be developed only after such a failure criterion has been established. Numerous failure criteria have been proposed in the past [2-6] for concrete or generally for brittle materials such as cast iron or rocks. These criteria can explain either the experimental data obtained in the biaxial tests or the data from the shear-axial stress tests. Very little effort, however, has been made which will lead to a systematic prediction of the results obtained from different types of test.

The failure modes of the specimens have never been successfully predicted by the existing failure criteria. In fact, a great deal of uncertainty still exists in experiments concerning the mode of failure. Fortunately, it is becoming clear that, after the efforts of many investigators, people are beginning to realize that the mode of failure can actually be controlled and that it depends greatly on the boundary conditions during experiments.

Concrete, as a continuum, behaves viscoelastically under long term-loadings. Under short-time load, however, concrete is quite elastic and may be assumed to obey Hooke's law.

In this paper, we propose a failure criterion for plain concrete which is based on the experimentally observed facts. It will be shown

*Micro-cracks in large number do exist before any load is applied [1].
that this failure criterion can successfully predict the experimental results obtained from the biaxial tests as well as those from the shear-axial stress tests. This criterion predicts, moreover, the modes of failure under all conditions.* Care must be exercised, however, to select experimental data to be used for comparison with the theory, since many experimental data are obtained under undesirable loading conditions.

* Failure due to buckling will not be discussed in this paper.
II. A FRACTURE-YIELD CRITERION

It is well-known that concrete may fail in shear under certain circumstances and fail in tension on other occasions. It is therefore appropriate to use a dual criterion of failure for concrete. This concept of a dual criterion was first suggested by Cowan [3], who assumed a cleavage fracture to be governed by the criterion of a constant maximum tensile stress or strain, and a shear failure to be governed by the maximum-shear theory advanced by Coulomb and Mohr.

While it is believed by the present author that any shear-type criterion, such as the theory of Coulomb and Mohr [2] or the theory making use of the octahedral shear stress and the mean normal stress [4], may describe the shear failure reasonably well, it is doubtful that any of the shear-type theories will provide a convincing criterion for cleavage fractures.

In the following, we shall propose for concrete a fracture-yield criterion, which is a dual criterion that is generally applicable to other isotropic brittle materials as well. No attempt will be made in this paper to propose a new yield criterion to govern the shear failure. It will be assumed that yielding is governed by any of the existing shear-type theories. We shall, however, investigate the cleavage fracture thoroughly, and a criterion for cleavage fracture will be proposed.

Before we proceed to the development of this new fracture criterion, we shall show that cleavage fractures are not macroscopically governed by stress but by strain. Therefore, we believe that a meaningful criterion for cleavage fractures must be based on strain. Let us consider, for instance, the unconfined compression test of rectangular prisms. For the sake of simplicity, we consider a plane-strain case. Figure 1 shows a specimen under uniform pressure p. Since cleavage fracture occurs at relatively small strain, it is appropriate to assume that Hooke's law applies in the derivation of a criterion for cleavage fracture. It is quite easy to show, using the theory of elasticity,
that the stress and the strain components corresponding to the plane problem in Fig. 1 are given by

\[
\begin{align*}
\sigma_x &= 0, \quad \sigma_y = -p, \quad \sigma_z = -\nu p \\
\tau_{xy} &= \tau_{yz} = \tau_{zx} = 0
\end{align*}
\] (2-1)

\[
\begin{align*}
\varepsilon_x &= \frac{\nu}{E}(1 + \nu)p, \quad \varepsilon_y = -\frac{p}{E}(1 - \nu^2), \quad \varepsilon_z = 0 \\
\gamma_{xy} &= \gamma_{yz} = \gamma_{zx} = 0
\end{align*}
\] (2-2)

where \(E\) and \(\nu\) are, respectively, Young's modulus and Poisson's ratio.

Experimental results show that cleavage fracture does occur in the specimen loaded as shown in Fig. 1, and that the cracks are parallel to the y-axis.* Equations (2-1), however, show that \(\sigma_x = 0\), i.e., cleavage fracture occurs in a direction perpendicular to the x-axis even though the normal stress is equal to zero in the x-direction. This finding is in direct contradiction to the concept that cleavage fractures are governed by stress. According to this concept, no cleavage fracture would occur with cracks parallel to the y-axis, since \(\sigma_x = 0\). We have thus seen that stress is not a convenient concept for the definition of the initiation of cleavage fractures.

From the viewpoint of continuum mechanics, we may define fracture as the separation of two neighboring particles in the continuum. As fracture occurs, the displacement between two neighboring particles suffers a jump. The displacement and the strain are, of course, related by the strain-displacement equations. Discontinuity in displacement will occur when the strain has reached a critical value. Strain may therefore be used as a quantity that governs the cleavage fracture, and the plane of fracture is normal to the direction of maximum tensile strain.

Generally, let us now propose that cleavage fracture occurs when a scalar-valued function of the strain tensor \(\varepsilon_{ij}\) reaches a critical value \(\kappa^2\), i.e., when

*We shall explore this point later in the text.
For isotropic materials, it can be shown [7] that Eq. (2-3) reduces to

\[ f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \kappa^2 \]  

(2-4)

where \(\varepsilon_1, \varepsilon_2, \text{ and } \varepsilon_3\) are the principle invariants of the strain tensor, and are given by

\[
\varepsilon_I = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \\
\varepsilon_{II} = \varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_3 + \varepsilon_3 \varepsilon_1 = \frac{1}{2}(\varepsilon_2^2 - \varepsilon_{II}) \\
\varepsilon_{III} = \varepsilon_1 \varepsilon_2 \varepsilon_3
\]

(2-5)

where \(\varepsilon_1, \varepsilon_2, \text{ and } \varepsilon_3\) are the principal strains, and \(\varepsilon_{II} = \sqrt{(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)}\) is the equivalent strain. Eq. (2-4) may therefore also be written as

\[ f(\varepsilon_I, \varepsilon_{II}, \varepsilon_{III}) = \kappa^2 \]  

(2-6)

It is seen that \(\varepsilon_I\) is of the order \(\varepsilon\), \(\varepsilon_{II}\) is of the order \(\varepsilon^2\) and \(\varepsilon_{III}\) is of the order \(\varepsilon^3\).

Now, since the strain \(\varepsilon\) is small, we may expand the function \(f\) of Eq. (2-6) in Taylor's series and neglect terms of order \(\varepsilon^3\) and higher. We thus obtain

\[
\frac{m}{E} \varepsilon_I + n \varepsilon_{II}^2 + \varepsilon_{II} = \kappa^2
\]

(2-7)

where \(m\) and \(n\) are material constants to be determined by experiments.

Eq. (2-7) is the criterion for cleavage fracture. According to this criterion, the occurrence of cleavage fractures is dependent upon an equivalent strain \(\sqrt[3]{\varepsilon_{II}}\) and also upon the volume change \(\varepsilon_I\) of the material element.

The plane of fracture is proposed to be normal to the direction of maximum tensile strain.
III. DETERMINATION OF MATERIAL CONSTANTS

To determine the constants \( m, n, \) and \( \kappa, \) three kinds of test are necessary. They are the uniaxial tension test, the unconfined compression test and the shear-axial stress test of a hollow thin-walled cylinder.

Referring to the principal coordinate system, Hooke’s law reads

\[
\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \\
\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_3 + \sigma_1)] \\
\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)]
\]

Substituting Eqs. (3-1) and (2-5) into (2-7) we obtain

\[
(1 + 2\nu^2)(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\nu(2 - \nu)(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) \\
+ \mu(1 - 2\nu)(\sigma_1 + \sigma_2 + \sigma_3) + \eta(1 - 2\nu)^2(\sigma_1 + \sigma_2 + \sigma_3)^2 = T^2
\]

where \( T = E\kappa. \) Eq. (3-2) is the criterion for cleavage fracture in terms of the principal stress components. Geometrically, Eq. (3-2) represents a surface in the principal-stress space. This surface will be termed the fracture surface in this paper.

The intercepts of this surface with the three stress axes are easily determined to be

\[
\sigma_1 = \sigma_T \text{ or } -\sigma_C \\
\sigma_2 = \sigma_T \text{ or } -\sigma_C \\
\sigma_3 = \sigma_T \text{ or } -\sigma_C
\]

where
\( \sigma_T = \frac{1}{2[(1 + 2v^2) + n(1 - 2v)^2]} \left\{ m(1 - 2v) \right. \\
+ \sqrt{m^2(1 - 2v)^2 + 4[(1 + 2v^2) + n(1 - 2v)^2]} T^2 \right\} \\
= \text{Intercept with the tension axis} > 0 \\
\sigma_C = \frac{1}{2[(1 + 2v^2) + n(1 - 2v)^2]} \left\{ m(1 - 2v) \right. \\
+ \sqrt{m^2(1 - 2v)^2 + 4[(1 + 2v^2) + n(1 - 2v)^2]} T^2 \right\} \\
= \text{Intercept with the compression axis} > 0 \quad (3-4) \\

The magnitude of \( \sigma_T \) and \( \sigma_C \) may be determined by using a simple tension and an unconfined compression test.

Now, let

\[ K = \frac{\sigma_C}{\sigma_T} \quad (3-5) \]

where \( K > 1 \) for brittle materials. Then, Eqs. (3-4) and (3-5) may be combined to obtain

\[ m = \frac{(K - 1)}{(1 - 2v)} T \sqrt{\frac{(1 + 2v^2) + n(1 - 2v)^2}{K}} \quad (3-6) \]

with \((1 + 2v^2) + n(1 - 2v)^2 \neq 0\). We note that \( m \) must be positive, since under hydrostatic compressive strain \( \varepsilon_I < 0 \) and if \( m < 0 \) then \( \frac{m}{E^I} > 0 \). This means that, referring to Eq. (2-7), cleavage fracture would occur when the equivalent strain \( \sqrt{\varepsilon_{II}^2} \) is small, which is contrary to the experimental findings.

Using (3-6), Eqs. (3-4) can now be simplified to obtain

\[ \sigma_T = \frac{1}{\sqrt{k[(1 + 2v^2) + n(1 - 2v)^2]} T} \]

and

\[ \sigma_C = \sqrt{\frac{K}{(1 + 2v^2) + n(1 - 2v)^2}} T \quad (3-7) \]
Eqs. (3-5) and (3-7) are sufficient for the determination of two constants. The third constant will be determined from the shear:axial-stress test.

Referring to a rectangular Cartesian coordinate system, the criterion for cleavage fracture (2-7) now becomes

\[(1 + 2v^2)(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - 2v(2 - v)(\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z)\]

\[+ 2(1 + v^2)(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) + m(1 - 2v)(\sigma_x + \sigma_y + \sigma_z)\]

\[+ n(1 - 2v)^2(\sigma_x + \sigma_y + \sigma_z)^2 = T^2 \quad (3-8)\]

which is a hyper surface in the nine-dimensional stress space. For the torsion-axial loading of the hollow thin-walled cylinders, the stress state is

\[
\begin{pmatrix}
\sigma & \tau & 0 \\
\tau & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\quad (3-9)
\]

where the x-axis is coincident with the axial direction of the cylinder; \(\sigma\) and \(\tau\) are, respectively, the normal and the shear stress acting on an element of the cylinder.

Under this loading condition, Eq. (3-8) reduces to

\[
[(1 + 2v^2) + n(1 - 2v)^2]\sigma^2 + m(1 - 2v)\sigma + 2(1 + v^2)\tau^2 = T^2 \quad (3-10)
\]

or, using Eqs. (3-6) and (3-7), we obtain

\[
\sigma^2 + (K - 1)\sigma_T \sigma + D^2 \tau^2 = K\sigma_T^2 \quad (3-11)
\]

where

\[
D = (1 + v)\sqrt{\frac{2}{(1 + 2v^2) + n(1 - 2v)^2}} \quad (3-12)
\]

Differentiating (3-11) with respect to \(\sigma\), we obtain
\[
\frac{d\tau}{d\sigma} = -\frac{1}{2D\sqrt{\tau}} \left[ 2\sigma + (K - 1)\sigma_0 \right]
\]  

(3-13)

which is expressed by

\[
\frac{d\tau}{d\sigma} = -\frac{K - 1}{2D\sqrt{K}} = -\ell
\]  

(3-14)

at the intercept of Eq. (3-11) with the \( \tau \)-axis. The slope \(-\ell\) may be determined by experiment. Eqs. (3-12) and (3-14) lead directly to

\[
n = \frac{1}{(1 - 2\nu)^2} \left[ \frac{8\kappa l^2(1 + \nu)^2}{(K - 1)^2} - (1 + 2\nu^2) \right]
\]  

(3-15)
IV. COMPARISON WITH EXPERIMENTAL RESULTS

The criterion for cleavage fracture proposed in Section II will now be applied to predict the experimental results obtained in the biaxial tests and in the shear-axial stress tests. Owing to the nonavailability in the literature of experimental results that use the same concrete mix in the two types of test mentioned above, we shall compare our theoretical predictions with the experimental data obtained by Kupfer, Hilsdorf and Rusch [8] in the biaxial tests and with those obtained by Bresler and Pister [4] in the shear-axial-stress tests. Although the concrete mixes used in these two papers are not the same, we feel that they do report typical test results obtained in the corresponding tests.

In the case of biaxial test, \( \sigma_3 = 0 \) and Eq. (3-2) reduces to

\[
\left[ (1 + 2v^2) + n(1 - 2v)^2 \right] (\sigma_1^2 + \sigma_2^2) - 2[\nu(2 - \nu) - n(1 - 2v)^2] \sigma_1 \sigma_2 \\
+ m(1 - 2v)(\sigma_1 + \sigma_2) = T^2 \tag{4-1}
\]

or, using Eqs. (3-6) and (3-7), we obtain

\[
\sigma_1^2 + \sigma_2^2 + 2B \sigma_1 \sigma_2 + (K - 1)\sigma_T(\sigma_1 + \sigma_2) = K\sigma_T^2 \tag{4-2}
\]

where

\[
B = \frac{n(1 - 2v)^2 - \nu(2 - \nu)}{n(1 - 2v)^2 + (1 + 2v^2)} \tag{4-3}
\]

From the data of Kupfer, Hilsdorf and Rusch [8] we found that \( K = 20^* \) for 4450-psi concrete. Using \( \nu = 0.2 \) and \( n = -2.63 \), Eq. (4-2) reduces to

\[
\sigma_1^2 + \sigma_2^2 - 19.62\sigma_1 \sigma_2 + 19\sigma_T(\sigma_1 + \sigma_2) = 20\sigma_T^2 \tag{4-4}
\]

The above equation has been plotted in Fig. 2. It is seen that the

*It is believed by many investigators that \( K = 9 \sim 15 \) [9]. This is, however, based on the compressive strength obtained under undesirable loading conditions. See later discussions in this section.
agreement with the experiment is very good.

In determining the value of \( K \), we must pay special attention to the fact that failure may be due either to shear or to cleavage fracture. In the two-dimensional case discussed above, the dual criterion of failure is represented by a closed curve in the \( \sigma_1 - \sigma_2 \) space (Fig. 3). This curve is formed by the intersection of the yield criterion and the criterion of cleavage fracture as described by Eq. (4-2). In the present discussion, for simplicity, the Coulomb-Mohr criterion is used to govern the shear failure. This criterion is represented by an irregular hexagon PNCDEQP in Fig. 3.

For a given mix of concrete—or, more generally, for a given brittle material—the criterion of cleavage fracture may be affected by the size and shape of the specimens. Therefore, the curves HBAGFJ, CAGE and LIAGKM are all reasonable candidates for a correct criterion of cleavage fracture, and the fracture-yield criterion may be described by one of the following three curves, i.e., curve ABCDEFGA, curve ACDEGA or curve ALDMGA.

The fact that the ratio \( K \) is dependent upon the shape of the specimens has not as yet been generally recognized, although the available data in the literature show such a dependency. More experimental work needs to be done in order to establish a functional relationship between \( K \) and a slenderness ratio \( q \) which is conveniently defined by

\[
q = \frac{\text{maximum dimension}}{\text{minimum dimension}}
\]  

(4-5)

Thus, \( q = 1 \) for a cube, \( q > 1 \) for a prism, and \( q \) is quite large for a thin-walled hollow cylinder.*

For cubical specimens, \( K \) is relatively small and the fracture-yield criterion is represented by the curve ALDMGA, which is similar to the experimental curves by Vile [10]** and Taylor et al [11]. The stress state I in Fig. 3 for the unconfined compression test will therefore cause cleavage fracture with cracks parallel to the direction of load. The latter result is also in agreement with the experimental observation by Vile [10] and Mills

*The well-known result that the compressive strength decreases with increasing \( q \) is due to frictional restraint of samples, and the samples fail in shear. Under careful arrangement, the samples will fail in tension and the compressive strength should increase with \( q \).

** Vile used specimens of 4x4x3-in. volume in the biaxial compression-tension tests.
and Zimmerman [12].

For specimens with large q such as long prisms and thin-walled hollow cylinders, shear failure prevails in the unconfined compression test [5], [6], [13], [14]. Although micro-cracks parallel to the direction of loading are observable in the specimens, because of the large height of the specimens failure can be delayed as long as the concrete is capable of withstanding internal crack formation. It takes a larger compressive load for the micro-cracks to link together and form macro-cracks which will lead to the final failure of the specimens. Therefore, the stress state H (Fig. 3) would be necessary for the specimens to fail in cleavage fracture. The maximum shear stress is, however, expressed by half the difference between the maximum and the minimum normal stresses. The minimum normal stress is equal to zero in the case of unconfined compression tests. As the maximum normal stress goes higher, it becomes possible that the maximum shear stress exceeds its yield limit, and the plastic yielding occurs before the specimen fails in cleavage fracture. The specimens failing under this condition will show, in addition to the familiar shear mode of failure, a laminated structure imparted to the concrete as a result of internal cracking in a plane parallel to the applied forces. This is so because maximum tensile strain occurs in the stress-free direction. The fracture-yield criterion of the aforementioned case is represented by the curve ABCDEFGA in Fig. 3. It is seen that the case of Fig. 2 corresponds to the present situation.

For not-so-brittle material, the distance NA in Fig. 3 is small. For ductile materials, the point A lies above the point N. In the latter case, the criterion for cleavage fracture does not intersect with the criterion for shear failure, and shear failure prevails under all loading conditions.

In the case of shear:axial-stress test, the cleavage fracture is governed by Eq. (3-11). Using the constants \( K = 20, \nu = 0.2 \) and \( n = -2.4 \), Eq. (3-10) reduces to

\[
\sigma^2 + 19\sigma_T\sigma + (3.65)^2 \tau^2 = 20\sigma_T^2
\]  

(4-6)

This curve has been plotted in Fig. 4. Without a doubt the agreement between theory and experiment is remarkable.
It will be noted that shear failure prevails when the hollow cylinder is under axial compression or when the specimen is subjected to combined compression and a very small amount of torsion [4]. The curve representing the criterion of shear failure is shown schematically in Fig. 4 using Coulomb-Mohr theory.

Some remarks are now in order. While the experimental results of Bresler and Pister [4] were obtained during shear: axial-stress tests, these data have repeatedly been replotted in the $\sigma_1 - \sigma_2$ space [8], [15]. This replotting of data is not legitimate, since various stress states in the $(\sigma_1, \sigma_2)$ space obtained do not correspond to the stress states of the same material element. Rotations of the principal axes are involved in this transformation.
V. THE MODES OF FAILURE

After tremendous efforts of many investigators, it is now believed that the discrepancies between test results from different sources can often be traced back to unintended differences in the stress states which have been developed in the test specimen. The most apparent discrepancy among the test results is the mode of failure, which—once considered to be intractable—is becoming predictable in experiments.

In the unconfined compression test of a cylinder or of a prism, the shear mode of failure is now believed to be a result of frictional restraint on the ends of the sample, and the true mode of failure is by axial cleavage [15]. In fact, a major improvement has been made in recent years regarding the application of brush bearing platens in the tests [8]. The frictional restraint has been drastically reduced through this method of testing, and it has been shown that the experimental results using brush bearing platens differ greatly from those using untreated bearing platens.

Further improvement in reducing the frictional restraint may be achieved by introducing two heads, one on top and one underneath the specimen, during compression tests. The two heads should be of the same material, width and thickness as the specimen. It was shown by Coker and Filon [16], using the technique of photoelasticity, that if the height of each head was greater than 0.4 times the height of the specimen, the stress distribution was almost uniform throughout the cross section of the specimen. This method has been proved to be very satisfactory in the testing of brittle materials such as ice at the Iowa Institute of Hydraulic Research.

The advantage for using this method of testing is that, when under compression, the material just above and below the contact surfaces between the heads and the specimen will expand the same amount, since they are of the same composition. The friction at the contact surfaces will thus be a minimum if not equal to zero. Moreover, the contact surfaces may be polished and lubricated to allow horizontal sliding motion of material elements which are adjacent to these contact surfaces, if the elements have a tendency to move. As a result of this arrangement, a uniform lateral expansion of the specimen can be achieved (Fig. 5a) as contrast to the bulged lateral surface under conventional method of testing (Fig 5b). The strain state
as well as the stress state will then be uniform throughout the specimen. It is felt that only the experimental data obtained under uniform stress and strain state are meaningful, if the results are to be used to define a stress-strain curve or to determine a failure criterion.

In the remaining part of this section, we shall discuss in detail the failure modes of various experiments reported in the literature in the light of the dual failure criterion proposed in this paper.

Unconfined compression tests of a cube, a short block or a cylinder have already been discussed earlier in this section. The failure mode in this case has been identified with the cleavage fracture. Tests on concrete plates have also been conducted by several experimenters [8], [17]. The failure modes observed by Kupfer et al. for loading conditions ranging from biaxial tension to biaxial compression can all be satisfactorily predicted by the present theory. In particular, failure is due to shear under uniaxial* and biaxial compressive loading. For the biaxial compression case, failure as observed by Liu et al. [17] occurred by splitting along planes parallel to the load and parallel to the face of the specimen. This result would seem to be in contradiction with the observation of Kupfer et al. This discrepancy can however be explained. The difference in the concrete mixes in the two papers is the main factor causing this discrepancy. In fact, the criterion of cleavage fracture as given by Eq. (4-2) becomes an equation of an ellipse if the constants K and n are properly adjusted. In the case under discussion the ellipse remains all the way inside the criterion of shear failure, so that cleavage fracture occurs even in the biaxial compression case. The planes of cracks are parallel to the face of the specimen, as the tensile strain is maximum in the stress-free direction. Finally, we note that this type of crack was also observable in the specimen of Kupfer et al. when subjected to biaxial compressive loading, even though the final failure was due to shear.

In the biaxial testing of thin-walled cylinders [13] [14] [6], the modes of failure can also be predicted by the present theory. The failure criterion of this case corresponds to the case expressed by curve ABCDEFGA in Fig. 3. Therefore, both cleavage and shear failure are expected to occur at their corresponding loading conditions. Under horizontal uniaxial com-

*See discussion of section IV and Figs. 2 and 3.
pression, maximum tensile strain occurs in the longitudinal direction of the cylinder. The specimen will therefore fail by splitting in a horizontal section and will be divided into two cylindrical pieces. Under biaxial compression, similar to the tests of concrete plates, the laminated structure imparted to the concrete has also been predicted by the theory, although the final failure may be due to shear.

The torsion-compression test of thin-walled cylinders involves only two stress and two strain components, namely, \( \sigma, \tau, \epsilon, \) and \( \gamma \). The other stress and strain components are negligibly small. With the direction of maximum tensile stress coincident with the direction of maximum tensile strain, the plane of fracture is therefore normal to the direction of the maximum tensile stress. Under pure torsion, the stress state of a material element is shown in Fig. 6 (a). The jagged line in the figure indicates the plane of fracture, which is seen to be making an angle of \( 45^\circ \) with the vertical axis.* As the axial compressive stress is applied and increased, the angle of the crack to the longitudinal axis is decreased; this is shown in Fig. 6 (b) and is in good agreement with the experimental observations by Bresler and Pister [14] and Goode and Helmy [18]. Some mixed-mode failures occurred in the experiments of Bresler and Pister. Under combined compression and torsion, two principal cracks were observed: a steep one inclined approximately 15 degrees to the longitudinal axis, and another making an angle of approximately 45 degrees with that axis. Our belief is that the former was due to shear failure which occurred when the axial compressive load was large**, and the latter was due to cleavage fracture. This type of shear failure has been discussed by Paul [19] using the Coulomb-Mohr theory.

Finally, we would like to remark that the "shear strength" obtained in conventional torsion tests is actually the amount of torque required to cause cleavage fracture of a cylinder. This, of course, is not the real strength for shear failure. We therefore propose that the shear strength may be obtained by performing confined compression tests. In this type of test, failure is always due to shear, and a shear-type theory may be applied to obtain the shear strength.

*Although indicated by a jagged line in the figure, the fractured surface is quite smooth.

** This is consistent with our analysis in section IV.
VI. SUMMARY

In this paper, we have proposed a strain-based dual criterion to govern the failure of plain concrete. It has been shown that this criterion can predict with great accuracy the experimental data obtained in the biaxial tests as well as those in the shear:axial-stress tests. Moreover, we have shown that the modes of failure of plain concrete are predictable if the stress states of the samples are carefully controlled. A method of testing has also been proposed for further improvement in reducing the frictional restraint during unconfined compression tests.
VII. REFERENCES


Fig. 1 Unconfined compression of a prism
Data by Kupfer, Hilsdorf & Rusch (1969)

Equation (4-4)

\[ \sigma_1^2 + \sigma_2^2 - 19.62\sigma_1\sigma_2 + 19\sigma_T(\sigma_1 + \sigma_2) = 20\sigma_T^2 \]

--- Coulomb-Mohr Theory

Fig. 2 Concrete (4450 psi) under combined axial stresses
Fig. 3 The dual criterion of failure in plane stress
Test Data by Bresler & Pister

- 3000-psi concrete
- 4500-psi concrete
- 6000-psi concrete

Equation (4-6)
\[ \sigma^2 + 19\sigma_T \sigma + (3.65)^2 \tau^2 = 20\sigma_T^2 \]

--- Coulomb-Mohr Theory

Fig. 4 Concrete under combined shear and compressive stresses
Fig. 5  A new method for compression testing
Fig. 6 Stress states of an element of a thin-walled cylinder