

**Some Effects of Suspended Sediment
on Flow Characteristics**

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SOME EFFECTS OF SUSPENDED SEDIMENT ON FLOW CHARACTERISTICS

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INTRODUCTION

Despite the steady advance in our knowledge of the mechanics of sediment transportation in the last two decades, there is still no theory that is completely satisfactory for engineering purposes. With present theories it is not possible accurately to predict the load under equilibrium conditions, and problems concerned with non-equilibrium conditions can hardly be treated at all because of their extreme difficulty. For instance, the elementary aspect of resistance to flow of a sediment-carrying stream is complicated by the fact that not only does the boundary configuration change with sediment transportation rate, but so does the internal turbulence, momentum transfer, and hence the shear. Once it is realized how long it has taken to bring the much simpler problem of resistance of clear fluids to the present state of knowledge, it is not surprising that progress on the sediment problem is not more rapid.

The writer believes that progress with the transport problem will be made by studying the effects of sediment on the flow characteristics, since ultimately transport must be expressed in terms of the flow parameters. Accordingly, the present paper will attempt to clarify the nature of some of the effects of sediment on the transporting flows. Some unpublished data will be introduced and discussed along with published results.

EFFECT OF SEDIMENT ON RESISTANCE TO FLOW

The resistance for steady uniform flow in a conduit can be expressed in terms of the friction factor f defined by the Darcy-Weisbach formula,

$$h_f = f \frac{L \bar{U}^2}{4R 2g}$$

in which h_f is the drop in piezometric head in a distance L , R is the hydraulic radius, g is the acceleration of gravity, and \bar{U} is the mean velocity. For sediment-free fluids f is a function of the Reynolds number and the roughness of the conduit boundaries, and all observations can be plotted on the familiar Stanton diagram. If sediment is being transported, the configuration of the bottom, and hence the roughness, changes, and in addition there appears to be an internal effect within the fluid due to the presence of suspended sediment.

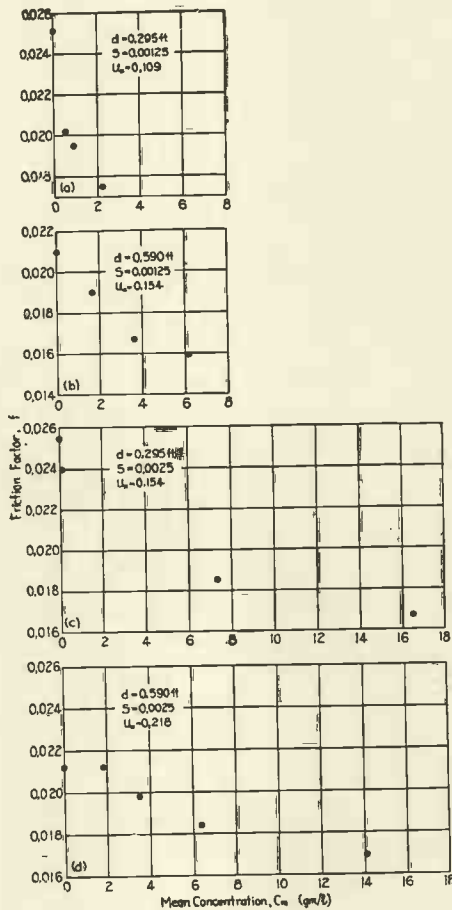


FIG. 1. FRICTION FACTOR OF A FLUME 33 IN. WIDE AS A FUNCTION OF CONCENTRATION OF 0.10 MM SAND IN THE FLOW.

In order to analyze the effects of sediment concentration, sets of experiments were conducted in which the hydraulic slope and hydraulic radius were held constant and the amount of sediment available for the flow to transport was varied. Since only the average concentration was varied, while the other quantities were kept constant, the effects of the sediment on the flow could be observed directly. The experiments were conducted by the author in a rectangular flume 33 in. wide and 60 ft. long, which is described in detail in a previous publication [1]. Friction factors obtained are shown in Fig. 1. The bed of the flume was roughened with 0.88-mm sand cemented to the bottom and the sides were of painted hot-rolled steel. The sand being transported was quartz with a geometric mean sieve size of 0.091 mm and a geometric standard deviation of 1.15, and is referred to as having a nominal size of 0.1 mm. The mean concentration C_m was measured by sampling the flow after it had discharged from the flume into a tank just ahead of the circulating pump. At this point all of the load, including that moving at or near the bed, was suspended in the flow and was sampled. By far the greater portion of the sediment moving in the flume was in suspension so that little error is introduced in considering C_m to be the suspended load.

It is clearly seen from Fig. 1 that the friction factor diminishes as the concentration C_m , or the rate of sediment transport, increases. Since in each of the four series of experiments the depth and slope were kept constant, the decrease in f was easily detected by an increase in the flow rate or average velocity.

Similar experiments were made by Ismail [2] in a rectangular pipe 10.5 in. wide by 3 in. high. Friction factors from experiments for two sands and four hydraulic gradients are shown in Fig. 2. The hydraulic gradient varied slightly within a set of runs as shown by the extreme values appearing in the figure. It will be seen that the friction factor tends to remain constant or to increase with sediment concentration, but, contrary to the results of Fig. 1, it shows a slight tendency to decrease. The increase in friction factor observed by Ismail can be explained as an effect of the dunes that form on the bed. He described the bed conditions as observed during his experiments, but in the majority of cases the concentration of suspended sediment was so high that the bed was obscured. However, in the lowest rates of flow, Figs. 2a and 2e, for which he was able to see the bed, he noted that dunes formed. It will be seen that

these are the cases showing most marked increase in f . At higher rates of flow, for which the friction factor did not change appreciably with increase in sediment load, it is probable that if dunes did exist on the bed, they were smaller than for the lowest flow since, as described by Gilbert [3], dunes which form at one velocity tend to be washed out or reduced in amplitude at some higher velocity.

In the flume experiments from which the data of Fig. 1 are taken, dunes were also observed to form, but unfortunately no detailed systematic observations were made of them. The height of the high-

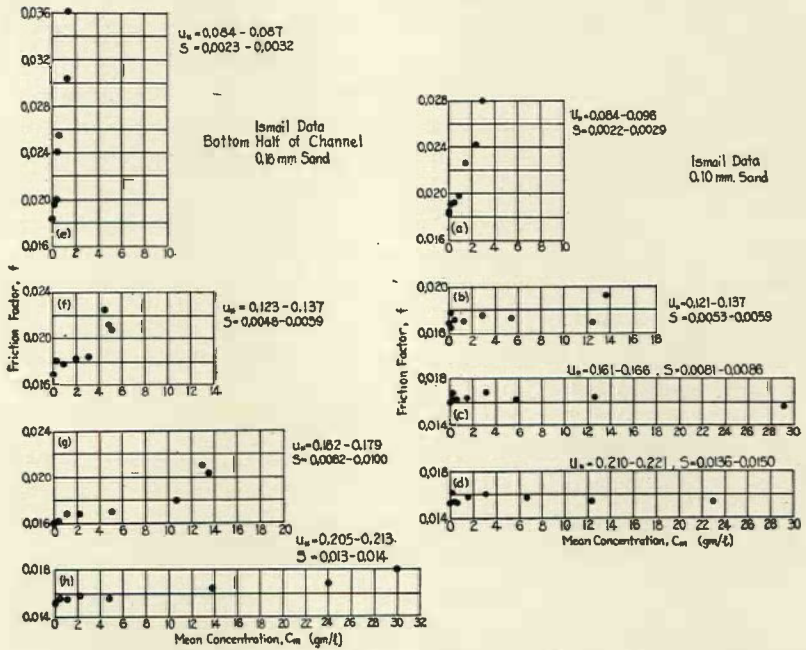


FIG. 2. FRICTION FACTOR OF A RECTANGULAR PIPE 3 IN. X 10-1/2 IN. AS A FUNCTION OF CONCENTRATION OF SAND IN THE FLOW.

est dune observed was about one-twentieth the water depth, and its length from crest to crest was about 0.8 the depth. Figure 1 shows that, despite the occasional presence of dunes, the friction factors for sediment-laden flows were always less than for clear flows. Evidently, then, there is some mechanism present by which the sediment reduces the resistance to flow. In these experiments it was sufficiently powerful to overcome even the effect of dunes in pro-

ducing roughness. Ismail and the writer have shown that the velocity gradient increases as the suspended sediment load increases. This can be brought out clearly by studying the velocity profiles of the flows. The logarithmic velocity distribution law is given by

$$\frac{U - U_{\max}}{U_*} = \frac{2.3}{k} \log \frac{y}{y_m} \quad (1)$$

in which U and U_{\max} are respectively the velocities at distances of y and y_m from the bed, U_* is the friction velocity or $\sqrt{\tau_0/\rho}$ in which

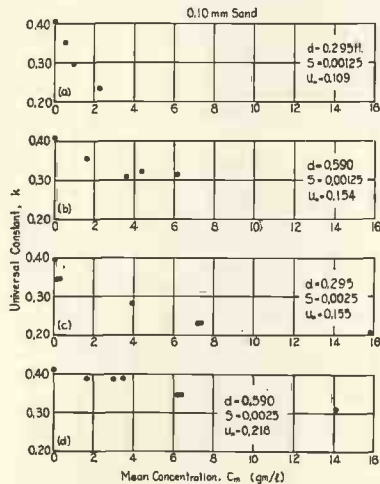


FIG. 3. VARIATION OF THE VON KÁRMÁN UNIVERSAL CONSTANT k WITH CONCENTRATION OF SAND IN THE FLOW IN A FLUME 33 IN. WIDE.

τ_0 is the shear stress at the bed and ρ is the mass density of the fluid, and k is the von Kármán universal constant. In an open channel y_m is the depth of the flow, in a rectangular pipe it is the distance from the bottom to the filament having the maximum velocity. This equation has been found to fit measured velocity profiles of sediment-laden flows very well and will be used here to discuss observed results. From Eq. (1) is obtained

$$\frac{dU}{dy} = \frac{U_*}{ky} \quad (2)$$

In the sets of experiments for which the hydraulic slope and the depth, and hence U_* , are constant, Eq. (2) shows that the velocity gradient is inversely proportional to k . In Fig. 3, k has been plotted against C_m for the four sets of experiments made in the 33-in. flume

described previously. The value of k is obtained from the slope of the graph of the measured velocity U against $\log y$ for the velocity profile at the center of the channel and the shear stress at the center, assuming two-dimensional flow with depth y_m . Figure 4 shows similar data taken from the experiments of Ismail for the bottom half of the channel and for sands with grain sizes of 0.10 and 0.16 mm. It will be seen that in all of the twelve sets of experiments represented by Figs. 3 and 4, the value of k diminishes as the average concentration C_m increases. From Eq. (2) it can be seen that this means that the velocity gradient dU/dy increases as the concentration increases.

The shear stress τ at any level in a two-dimensional flow can be expressed by the Boussinesq equation

$$\tau = \rho \epsilon_m \frac{dU}{dy} \quad (3)$$

in which ϵ_m is the coefficient for momentum exchange, also sometimes called the eddy viscosity. In any of the sets of tests described above, the shear at any distance y from the bed is essentially constant for the set. Therefore, an increase in dU/dy means that there has been a decrease in the exchange coefficient ϵ_m . This effect has

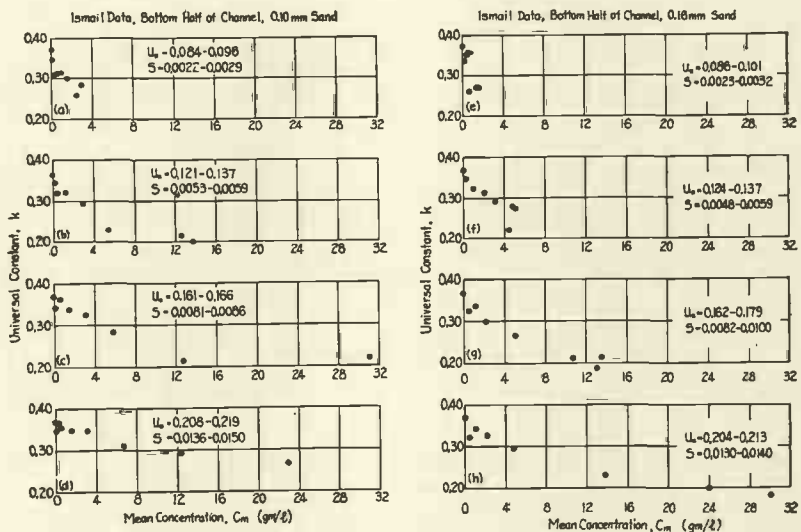


FIG. 4. VARIATION OF THE VON KÁRMÁN UNIVERSAL CONSTANT k WITH CONCENTRATION OF SAND IN THE FLOW IN A RECTANGULAR PIPE 3 IN. X $10\frac{1}{2}$ IN.

been ascribed to damping of the turbulence by the suspended sediment [1].

The data presented above show that, if sediment is being transported in suspension in flows with a given hydraulic gradient and depth, the velocity gradient increases with the sediment load. The data also show that the friction factors of the flow may increase or decrease as the sediment concentration increases. Using the relationships for pipes [4] and making appropriate changes, the friction factor for a two-dimensional channel can be expressed as

$$\sqrt{8/f} = A_r + \frac{2.3}{k} \log \frac{y_m}{Dk_s} \quad (4)$$

in which D is a constant, k_s is the equivalent sand roughness of the walls, and A_r is an empirical function, shown in Fig. 5, plotted as

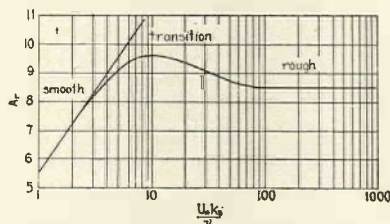


FIG. 5. THE COEFFICIENT A_r FOR ROUGH SURFACES AS A FUNCTION OF THE ROUGHNESS CHARACTERISTIC.

a function of the roughness characteristic U_*k_s/ν using data of Nikuradse [5]. As indicated in the figure, A_r is given over the range from hydrodynamically smooth to rough surfaces. The above results are for infinitely wide channels roughened with uniform sands of size k_s . However, as shown by Keulegan [6], this same equation can be used for rectangular channels by a proper adjustment in the constant D . If the walls are rough, A_r is constant and Eq. (4) shows that f will decrease as both k and k_s decrease. In the flume tests (Figs. 1 and 3), the bottom, which consisted of 0.88-mm sand, was on the verge of being completely rough, having values of the roughness characteristic ranging from 30 to 60. If the bed were covered with a smooth layer of the 0.1-mm sand being transported, the roughness characteristic would range from 3.5 to 7.0 and A_r could increase from about 8.5 to 9.6, causing some decrease in f . However, since the increase in A_r is brought about by a decrease in k_s , and since k was also observed to decrease (Fig. 3), all factors

would act to decrease f . In some cases dunes were formed on the bed and must have increased the equivalent roughness k_s above that of the 0.88-mm sand cemented to the bottom, thus tending to increase f . In such cases the effect of the diminished k must have been greater than that of k_s , since the net effect was to reduce f . In the experiments of Ismail (Fig. 2) for which f either increased or stayed about constant, the bed roughness k_s must have increased sufficiently to outweigh the effect of the decrease in k .

It has been suggested [7] that the shape of the velocity profile may be affected by the boundary roughness. If this is true, from Eq. (1) one would expect k to vary with roughness. In order to explore this point, the data of Nikuradse [5] on rough pipes were analyzed. This showed that the values of k , the von Kármán universal constant, did not vary appreciably over the range of these experiments in which the relative roughness r/k_s varied from 507 to 15, r being the pipe radius and k_s the sand size used to roughen the pipes. The minimum value of k was 0.324 and occurred for a relative roughness of 507, the next smallest value was 0.342. The maximum was 0.415 and the average of all experiments was 0.374. This is less than the average value of 0.4 used by Nikuradse and may be due to the slightly different approach to fitting the curves to the data. Average values of k for clear water obtained by Ismail in a rectangular pipe and the writer in a flume, are 0.372 and 0.407, respectively.

It appears from the above that k , and hence the shape of the velocity profile, does not vary appreciably for clear fluids and that assuming it to be constant is reasonable. The minimum value of k obtained from the Nikuradse data was 0.324, whereas with suspended load, as shown by Figs. 3 and 4, values of k less than 0.3 were obtained frequently and several values fell below 0.2. In view of this evidence, it seems safe to conclude that suspended load is a major factor in reducing k , which leads immediately to the conclusion that for a given value of the shear stress at a point in a stream, the velocity gradient increases as the suspended load increases.

INTERPRETATION OF OBSERVATIONS OF RESISTANCE AND VELOCITY

The observations of resistance and velocity referred to above can be explained, as was done by Ismail [2], in terms of the change in configuration of the bed and of the action of the suspended sediment on the turbulence. The formation of dunes on the bed increases

the roughness and tends to increase the friction factor for the flow. Since the dunes can vary widely from large amplitudes to essentially zero, depending on the velocity, the bed roughness can vary over a wide range.

The effect of the suspended sediment is, as has been outlined previously [1], a damping of the turbulence since the energy to suspend the sediment comes from the turbulence. Following Einstein and Chien [8], one can express the energy per unit width and unit time P_s required to suspend the sediment as

$$P_s = \left(1 - \frac{\gamma_w}{\gamma_s}\right) y_m \Sigma \bar{C} w$$

and the energy of friction P_f , lost by the flow per unit width and unit time as,

$$P_f = \gamma_m y_m \bar{U} S$$

in which γ_w and γ_s are the specific weights of the water and sediment, respectively, \bar{C} is the average concentration of sediment over the depth in weight per unit volume for the size fraction having a settling velocity w in still water, y_m , \bar{U} and S are the depth of flow, the average velocity, and the slope, as before, and the sum indicated is taken over all values of w . The ratio of these two quantities is

$$\frac{P_s}{P_f} = \left(1 - \frac{\gamma_w}{\gamma_s}\right) \frac{\Sigma \bar{C} w}{\bar{U} S} \tag{5}$$

Figure 6 is a graph of k plotted against the ratio of P_s to P_f for the data of Fig. 3. In calculating the abscissa values, \bar{C} has been taken as the average concentration over the upper 95 percent of the center profile of the flume and \bar{U} has been taken as the average velocity at this profile. The data of Fig. 6 correlate in a qualitative way

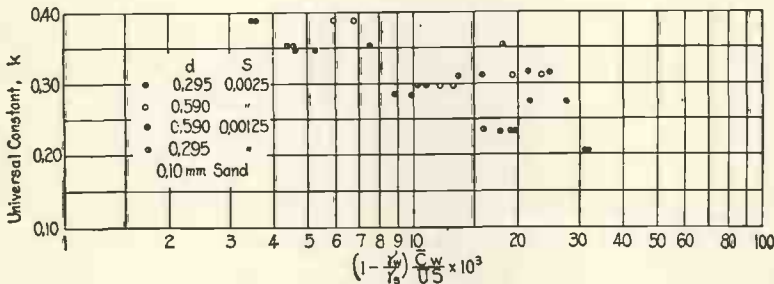


FIG. 6. THE VON KÁRMÁN CONSTANT k PLOTTED AGAINST THE RATIO OF POWER TO SUSPEND SEDIMENT AND POWER TO OVERCOME CHANNEL FRICTION.

but do not define a functional relationship. Einstein and Chien obtained somewhat similar results from plotting the data of Fig. 4 and data obtained on the Missouri River at Omaha.

The range in the ratio P_s/P_f in Fig. 6 is 0.003 to 0.03, that is, the power to support the sand varies from 0.003 to 0.03 of that required to overcome the fluid resistance of the stream. The power to support the sediment seems comparatively small and, in view of

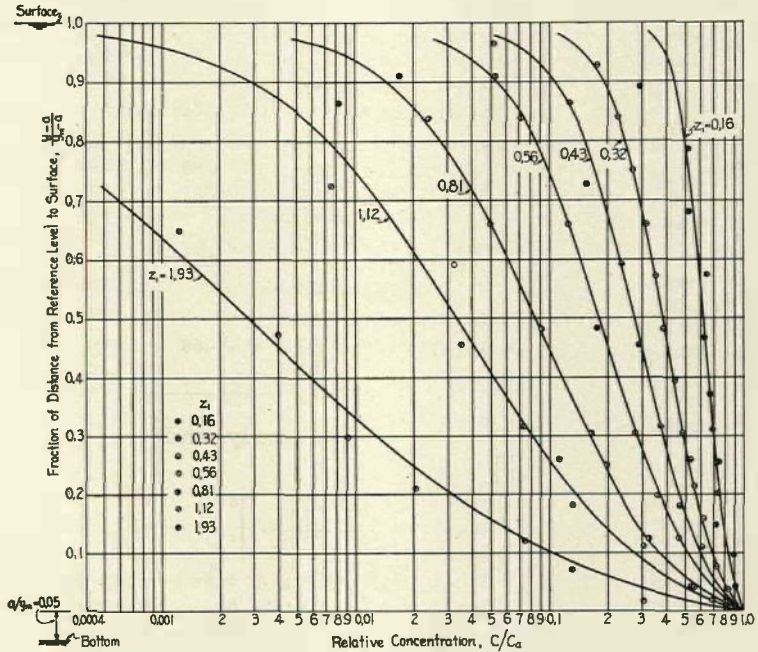


FIG. 7. VERTICAL DISTRIBUTION OF RELATIVE CONCENTRATION OF SEDIMENT OVER A WIDE RANGE OF CONDITIONS OF FLOW AND SEDIMENT SIZE.

this, it is surprising that the sediment can have such a large effect. In lectures on turbulence Prof. J. M. Burgers has suggested that the turbulence components which are effective in the transfer process may represent only a small part of the total turbulence energy; and since the energy to support the sediment must come from these components, even the withdrawal of the small energy to support the sediment can result in appreciable changes.

DISTRIBUTION OF SUSPENDED SEDIMENT

The equation for the distribution of suspended sediment with

TABLE I
DATA FOR CURVES OF FIG. 7

Exponent z_1	Flow depth y_m ft.	Slope ft. per ft.	Size of suspended sand mm.	Conc. at reference level $y = .05 y_m$ gm/liter	Site of measure- ments
0.16	9.9	.000125	.044 to .062	0.134	Missouri River at Omaha 10-18-51
.32	.590	.00125	.10 .062	6.95	Laboratory Missouri River
.43	7.7	.000121	to .074	.240	at Omaha 10-17-51
.56	.295	.00125	.10	3.20	Laboratory
.81	.295	.00125	.10	17.0	Laboratory
1.12	7.7	.000121	.149 to .210	2.53	Missouri River at Omaha 10-17-51
1.93	.600	.00125	.195 to .208	.56	Laboratory

settling velocity w in a two-dimensional, steady, uniform flow, which was first given by Rouse [9], is

$$\frac{C}{C_a} = \left[\frac{y_m - y}{y} \frac{a}{y_m - a} \right]^z \quad (6)$$

in which C and C_a are the concentrations at elevation y and a , respectively, above the bed, y_m is the depth of flow and the exponent is

$$z = \frac{w}{k\sqrt{\tau_a/\rho}} \quad (7)$$

the symbols being as defined previously. The form of Eq. (6) has been found to apply to laboratory studies in a flume, a rectangular pipe [2], and in a natural stream [10]. However, the exponent z that fits the observations is usually different from that given by Eq. (7). The studies of the problem made thus far have not supplied an expression for the exponent that agrees with experiment.

Figure 7 shows measured sediment distribution plotted against $\frac{y-a}{y_m-a}$. The curves represent the relative concentration C/C_a calculated from Eq. (6) using empirical values z_1 of the exponent. The

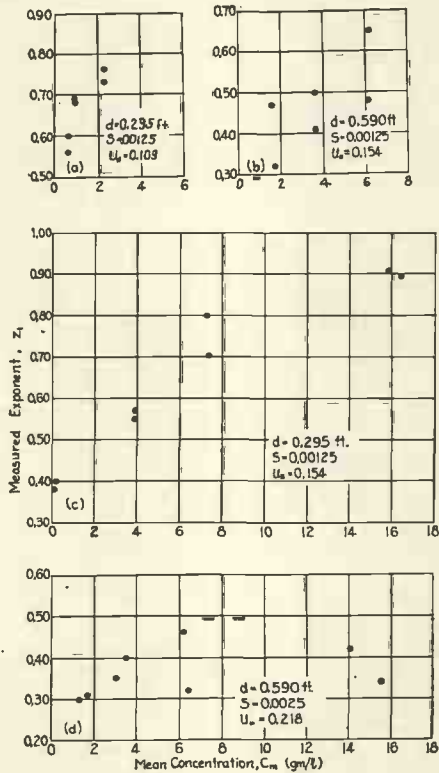


FIG. 8. VARIATION OF THE EXPONENT z_1 WITH CONCENTRATION OF SEDIMENT BEING TRANSPORTED IN A FLUME 33 IN. WIDE.

measurements were made in the laboratory flume used to obtain the data of Fig. 3 and in the Missouri River at Omaha [11] under conditions listed in Table I. It is seen that the data follow the form of Eq. (6) very well over the extreme range of conditions from a laboratory flume with flows a few inches deep to a large stream with flows almost ten feet deep.

Figure 8 shows z_1 plotted against the mean concentration for each of the four sets of tests in the 33-in. flume, results of which are also presented in Figs. 1 and 3. Figure 9 shows similar data from tests by Ismail in a rectangular pipe 3 x 10.5 in. Figures 8a, 8b, and 8c indicate that z_1 increases with concentration although there is considerable scatter in the results and no function is delineated. Figure 8d indicates no significant change in z_1 with C_m . Of the eight

sets of data of Fig. 9, all except the one of Fig. 9a show an increase in z_1 with C_m . Since the results of ten of the twelve sets of experiments shown in Figs. 8 and 9 indicate that z_1 increases with C_m , it seems safe to conclude that this is a general tendency. An increase in z_1 is to be expected from Eq. (7) since, as was shown above, k decreases with concentration. Inspection of Fig. 7 will show that as the exponent z_1 increases the concentration becomes less uniformly distributed over the depth or that dC/dy increases.

The differential equation for the distribution of suspended material is given by

$$wC + \epsilon_s \frac{dC}{dy} = 0 \quad (8)$$

in which ϵ_s is the exchange or diffusion coefficient for sediment. Solving Eq. (8) for ϵ_s and introducing Eq. (6) gives

$$\epsilon_s = \frac{wy(y_m - y)}{z_1 y_m} \quad (9)$$

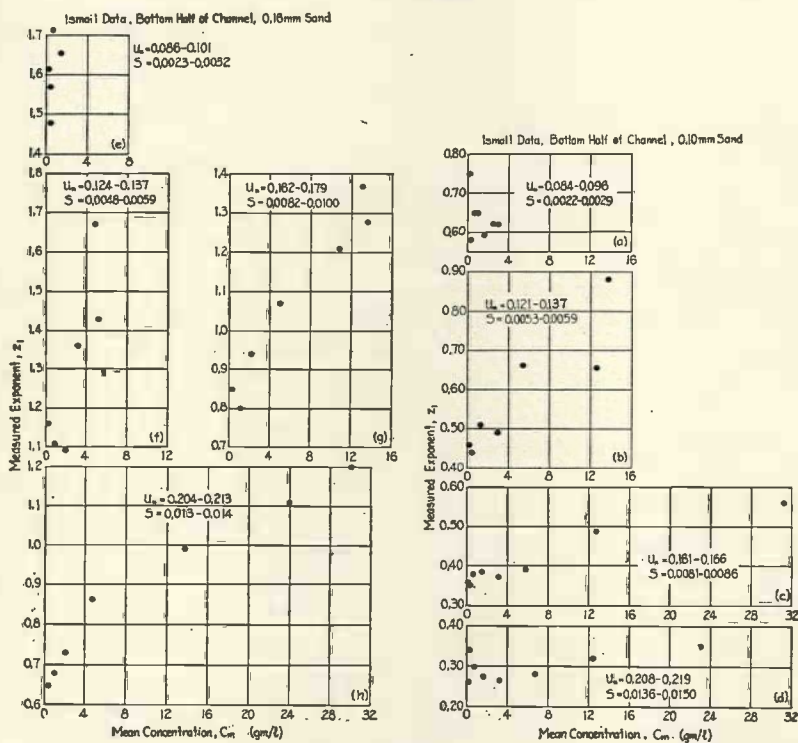


FIG. 9. VARIATION OF THE EXPONENT z_1 WITH CONCENTRATION OF SEDIMENT BEING TRANSPORTED IN A RECTANGULAR PIPE 3 IN. X $10^{-1/2}$ IN.

This shows that for given values of w , y , and y_m the exchange coefficient ϵ_s varies inversely as the exponent z_1 . Since in each of the twelve sets of runs upon which Figs. 8 and 9 are based, the size of sediment and the depth were kept the same, one can conclude that the exchange coefficient was reduced as the concentration increased. The above analysis assumes that w does not change with concentration which, as pointed out by Laursen and Lin [7], is in error. The highest concentration attained in the experiments was about 30 grams per liter, which according to Laursen and Lin decreases w by about 20%. A decrease in w will actually decrease z and cannot explain the observed increases in z shown in Figs. 8 and 9.

The exchange coefficients ϵ_m and ϵ_s which are defined by Eqs. (3) and (8) are related closely although there is not complete agreement on their exact relationship. Theoretical consideration by von Kármán [12] and Burgers [13] indicates that the coefficients need not be the same; studies by Carstens [14] lead to the conclusion that ϵ_s can never exceed ϵ_m . The writer found that the ratio of these coefficients could be greater or less than unity, Ismail found that ϵ_s was larger than ϵ_m and Laursen and Lin concluded that the two coefficients were the same. Nevertheless, the form of the two functions seems to be about the same. The data presented in Figs. 3 and 4 show that k , and hence ϵ_m , decreases as the concentration increases. Figures 8 and 9 show that z_1 increases with the concentration, which according to Eq. (9) means that ϵ_s decreases. From this it seems reasonable to conclude that the coefficient ϵ_s like ϵ_m is reduced by the damping effect of the suspended sediment on the turbulence. This is in disagreement with Laursen and Lin who conclude the sediment has little or no effect on the flow.

CONCLUSION

On the basis of experiments by the writer and others, it is concluded that suspended sediment damps the turbulence of the flow in such a manner that both the exchange coefficients, ϵ_m for momentum and ϵ_s for sediment, are reduced as the sediment concentration is increased. The friction factor of flow tends to be reduced when ϵ_m is reduced but is increased by dunes forming at the bed. This means that the friction factor of sediment-laden flow may increase or decrease with sediment load depending on which of these factors is the larger.

The accepted theory of suspended load, while being very useful

in analyzing sediment transportation problems, is inadequate to account for the effects of the sediment on the flow.

ACKNOWLEDGMENTS

The author is grateful to the Missouri River Division of the Corps of Engineers for permission to use field data included in Fig. 7.

DISCUSSION

Mr. Mitchell pointed out some of the difficulties in applying the results of laboratory studies to the investigation of sediment transportation of natural streams, particularly that of the Missouri River, now being made by the Omaha District of the Corps of Engineers. The development of a suitable engineering procedure for computing the total sand transport of a stream under nearly any ordinary set of hydraulic conditions has necessitated the consideration of transport of bed material along the stream bed as well as in suspension. The work has been largely directed toward verification of the applicability of basic fluid mechanics relations, such as Professor Vanoni has used, to specific conditions in a study reach of the Missouri River at Omaha. Since most of these relations are applicable at present only to two-dimensional flow, measurements so far have been confined to single verticals at selected spots in a reach. Velocity and sediment concentration at successive depths from the water surface have been measured simultaneously. Water-surface slopes have been obtained from readings of staff gages along the shores.

A number of maps have been made by fathometer of the river bed in the upstream 3,000 feet of the study reach. Different bed conditions correspond to significantly different values of Manning's n . Most of the velocity and sediment measurements have been taken at single verticals located in a comparatively flat area in the left half of the channel. The total suspended sediment concentration of the Missouri River on the days when measurements were made ranged from about 8 down to 3 grams per liter (8000 to 3000 ppm.). The sands constituted 30 percent or less of this concentration.

From the equation

$$k = 2.3 \frac{u^*}{N}$$

in which $u^* = (gdS)^{1/2}$ and N is the slope of semi-logarithmic plot of the measured velocity distribution, a plot was made of k against

concentration (Fig. 10). No particular trend is apparent. However, a plot of z_1 , the slope of the logarithmic plot of the measured distribution of a certain size of sediment against concentration indicates a possible decrease in z_1 with increased concentration (if one questionable value is omitted) as well as with the anticipated decrease with decreasing grain size.

Although the measured sediment distributions are found to fit the familiar distribution law, considerable difficulty has been found in obtaining consistent values of N . Figure 11 shows the variation in both the velocity and sediment distributions on two successive days when the discharge was nearly the same. Although the point velocities scatter, the slopes of the velocity distributions are fairly

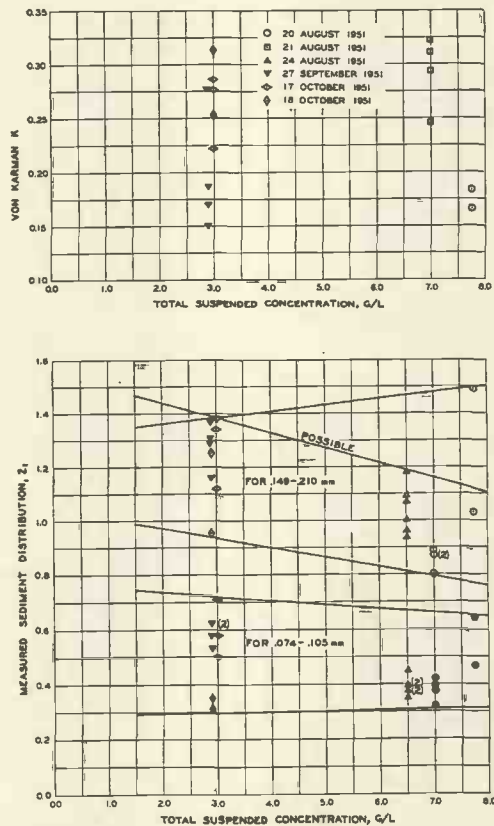


FIG. 10. VARIATION IN k AND z_1 WITH CONCENTRATION, MISSOURI RIVER AT OMAHA, NEBRASKA.

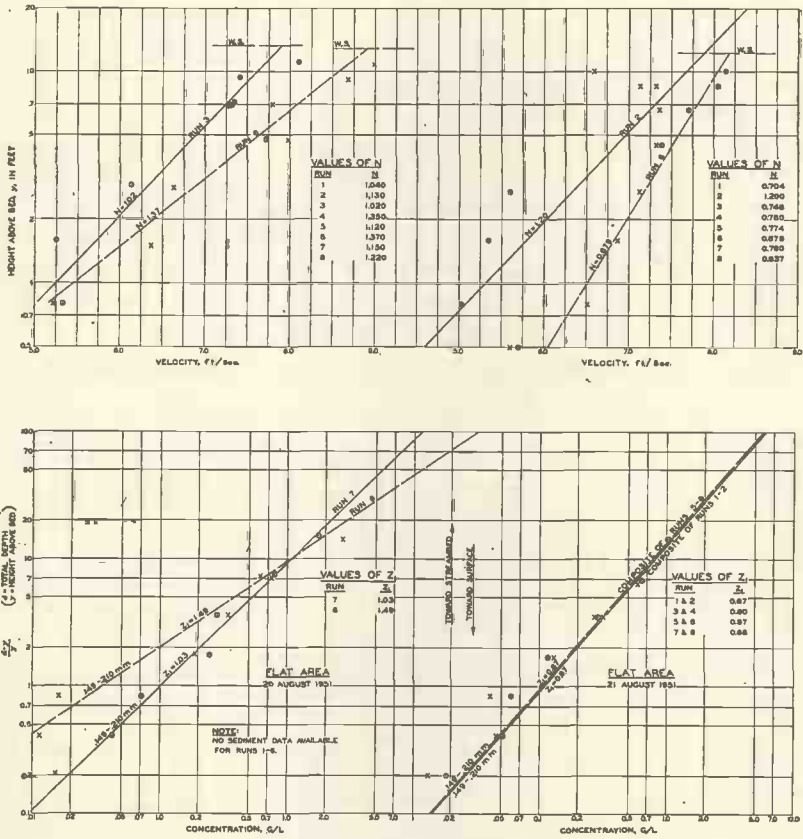


FIG. 11. VARIATION OF VELOCITY AND SEDIMENT DISTRIBUTION WITH VELOCITY CHANGES, MISSOURI RIVER AT OMAHA, NEBRASKA.

definite. The values of N , although fairly consistent for successive runs at the same point on each day, differ radically on the two days. Sediment data taken on 20 August were fragmentary and there is a question as to the validity of one of the two sediment distributions that were obtained. It has been found on other days that there is good agreement in z_1 even if N varies widely. Theory gives two relations between u_* and k in terms of easily measured parameters

$$ku_* = \frac{w}{z_1} \text{ and } \frac{u_*}{k} = \frac{N}{2.3}$$

The actual nature of these relations for various flow conditions has

been and apparently will continue for some time to be the major problem in field studies of sediment movement.

An example of the difference in vertical velocity and sediment distributions on hydraulically rough and smooth beds in the Missouri River may be interesting in light of Professor Vanoni's discussion of frictional resistance. On 18 October, measurements were made for such a comparison. Plots of the vertical distributions, shown on Fig. 12, indicate an increased degree of turbulence in an area in which sand waves occurred over that in a flat area by the greater irregularity of the velocity distributions and a lower value of N , but a more-uniform distribution of different sizes of sediment from bed to surface. It should be noted that there is a considerably lower

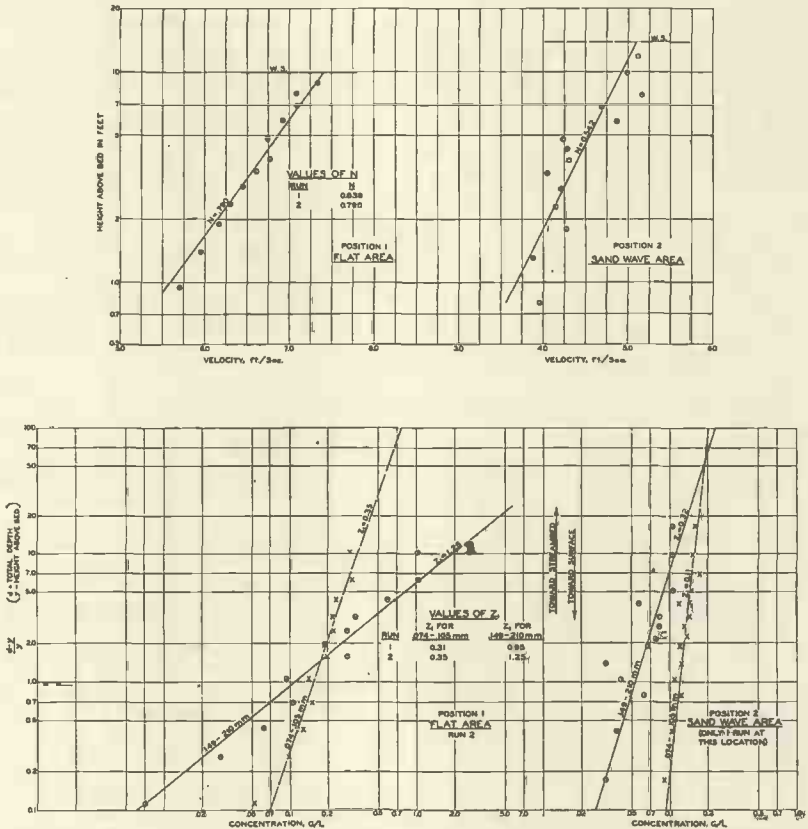


FIG. 12. VARIATION OF VELOCITY AND SEDIMENT DISTRIBUTION WITH BED FORM, MISSOURI RIVER AT OMAHA, NEBRASKA.

velocity and less sediment movement in the sand wave area than there is in the flat area less than 200 feet away.

In a forthcoming paper on this subject Mr. Mitchell hopes to present more data to assist in the evaluation of suspended sediment transport theory in the field.

Mr. Laursen stated that experiments at the Iowa Institute on the transport of suspended sediment have produced data which in general are similar to those presented. The interpretation which has been given those data, however, is quite different. One important difference in the conduct of the experiments should be mentioned. The experiments at the Iowa Institute have been conducted with a bed of sediment in the channel — this bed then being free to form dunes or ripples as conditions dictate.

Figure 13 shows the variation of f , k , and m (from $v=Cy^{1/m}$) as correlated with the concentration and the relative roughness. It is believed that the correlation of these quantities with relative roughness is one of cause and effect. Mr. Rand's experiments on the correlation of the variation of f and k with relative roughness mentioned in his discussion of Zingg's paper reinforce this conclusion, since his experiments were confined to clear water. The roughness elements used by Mr. Rand differed geometrically from the dune form, but the same trend is evident.

The apparent correlation of f , k , and m with concentration is believed to be the result of concentration varying with roughness. In

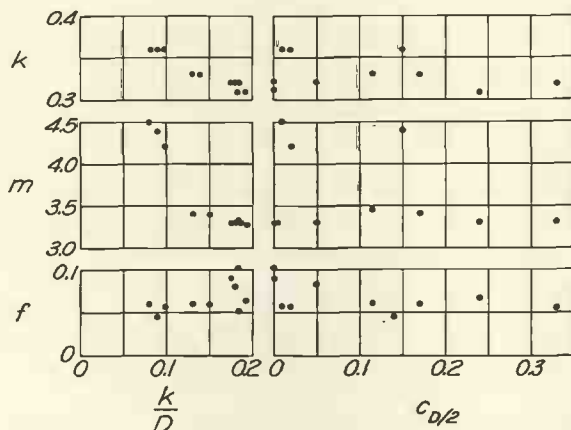


FIG. 13. RELATION OF CONCENTRATION WITH RELATIVE ROUGHNESS, FLUME MEASUREMENTS AT THE IOWA INSTITUTE OF HYDRAULIC RESEARCH.

fact, it is believed that roughness is one of the most important factors in the entrainment phenomenon. That roughness is not the only requirement for a high concentration is apparent from the two runs for zero concentration. These runs were made by forming a rough bed and then reducing the velocity until almost no sediment was in suspension. The f , k , and m values for these runs are ample evidence that relative roughness and not concentration is the cause of the variation of these quantities.

The Iowa experiments show ϵ_s/ϵ_m values greater than unity—agreeing with the data presented in Mr. Vanoni's paper. Even with the fall velocity corrected for the actual size of particles in suspension and for concentration, ϵ_s/ϵ_m is greater than unity. Moreover, the same values result whether the concentration distribution is derived from a logarithmic or an exponential velocity distribution.

Mr. Beckman remarked that alluvial streams, such as the Missouri and Mississippi Rivers, are subject to large shifts in the stage-discharge relation during changing flows. The U. S. Geological Survey has attempted to find the causes for such shifting, with little success. As a rule, the stage-discharge relation moves to the left during flood periods. It is not thought that change in slope could be the primary cause, as the streams cannot change slope appreciably from one stage to another. It must, then, be caused by either changes in the cross-sectional area, roughness of the bed, or both.

Mr. Rand (in referring to Mr. Laursen's discussion) said that since he worked with clear water, the variation of the k value could not be explained by the influence of suspended sediment. The k value proved to be a function of relative roughness and of the shape of the roughness. Since there is a great variation possible in the shape of roughness, a general law for variation of k with relative roughness has not been developed. It would also be very difficult to learn the constancy of the k -value by plotting the Nikuradse data, because for that study the relative roughness was low ($\bar{r}_0/k > 15$) and it is not generally possible to get much exactitude by reading the k value from semi-logarithmic plotting.

In concluding, Mr. Vanoni said that he was acquainted with the fine work that Mr. Mitchell and his associates are doing on the Missouri River and was glad to see the results obtained so far. These studies, which, as far as he knows, are the most detailed ever conducted on a major stream, are serving to confirm some of the relations developed in the laboratory and to indicate where such rela-

tions are not applicable. At the same time new information on the behavior of streams is being obtained which will serve to guide research in this field. Measurements on rivers, although very expensive, are necessary to test the results of small-scale laboratory experiments. It is through this process of laboratory developments and field testing that the means to predict the behavior of streams will gradually be attained. The fact that there is no particular correlation between concentration and k in Mr. Mitchell's measurements should not be taken as a final result, in view of the difficulties of obtaining consistent values of k in the laboratory as well.

The experimental results presented by Mr. Laursen showing relations between k , m and f and the relative roughness are very interesting, and it is hoped that he will report them more completely in the near future since they will help to clarify the problem of sediment movement. The strongest evidence to support his idea that relative roughness is one of the most important factors in entrainment is obtained from the interesting results of Mr. Rand showing that k , m , and f for clear water flows also correlate with relative roughness.

Figure 1 shows that it is possible to reduce f as the concentration increases, while at the same time (Fig. 3) the k values are decreasing. Equation (4) shows the relation between f , k , and k_s and indicates that an increase in k_s might be overshadowed by a decrease in k with the over-all result that f would decrease. In the experiments represented in Figs. 1 and 3, the relative roughness of the bed certainly did not increase for the low concentrations and probably actually decreased due to deposit of fine sand in the bed which was roughened with 0.88 mm sand. At the higher concentrations the bed roughness was increased by the formation of dunes, yet the friction factor still decreased. If, as Mr. Laursen suggests, the decrease in k is due to an increase in the relative roughness, then in this case one cannot explain a continuous decrease in k while the bed roughness first decreased and then increased. Therefore, it still seems reasonable to ascribe the change in k to the suspended sediment, although the writer agrees that other factors may be entering. As pointed out by Mr. Laursen, his experiments were made with the bed completely covered with sand, while in many of the experiments discussed in this paper very little sediment was on the bed. Although this difference could account for some difference in the results, it does not seem likely that it would account for the di-

versity of ideas on the explanation of the observed effects. A final resolution of the differences must wait until more complete data are available.

Mr. Beckman's account of his experiences with changes in stage-discharge relationships for rivers during floods is interesting and can very likely be explained, at least in part, by changes in bed roughness and internal effects as represented by the variation of k . A more complete discussion of this problem would be useful to workers in sediment transportation and general river regulation.

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