Prediction of the Damping-Controlled Response of Offshore Structures to Random Wave Excitation

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A method is presented for predicting the damping-controlled response of a structure at a known natural frequency to random wave forces. The principal advantage of the proposed method over those in current use is that the explicit calculation of wave forces is not required in the analysis. This is accomplished by application of the principal of reciprocity: that the linear wave force spectrum for a particular vibration mode is proportional to the radiation (wave-making) damping of that mode. Several example calculations are presented including the prediction of the heave response of a tension-leg platform. The directional distribution of the wave spectrum is included in the analysis.

INTRODUCTION

This paper introduces a simple procedure for estimating the dynamic response of a structure at each of its natural frequencies to the random excitation of ocean waves. The principal advantage of the proposed method is that the explicit calculation of wave forces has been eliminated from the analysis. This is made possible by a direct application of the reciprocity relations for ocean waves, originally established by Haskind[1] and described by Newman,[2] in a form that is easy to implement. Briefly stated, for many structures it is possible to derive a simple expression for the wave force spectrum in terms of the radiation damping and the prescribed wave amplitude spectrum. In general, such a substitution is of little use because the radiation damping coefficient may be equally difficult to find. However, the substitution leads to a very useful result when the dynamically amplified response at a natural frequency is of concern. In such cases it is shown that, contrary to popular belief, the response is not inversely proportional to the total damping but is, in fact, proportional to the ratio of the radiation damping to the total damping. This is because the radiation damping and the wave exciting forces are not independent quantities. Therefore, in the absence of a reliable estimate of either the total damping or the ratio of the radiation component to the total, an upper bound estimate of the response still may be achieved because the ratio is, at most, one. The demonstration of the existence of this upper bound is one of the key contributions of this paper.

Linear wave theory is assumed; therefore, excitation caused by drag forces is not considered. However, for many structures drag excitation is negligible except for very large wave events. In the design process extreme events are modeled deterministically by means of a prescribed design wave and not stochastically as is done here. In many circumstances linear wave forces will dominate, and the results shown here will be applicable. Although drag-exciting forces are not included, damping resulting from hydrodynamic drag is included. Wave diffraction effects are extremely difficult to calculate. This analysis includes effects but never requires explicit evaluation of them.

It has been recognized that directional spreading of the wave spectrum is an important consideration in the estimation of dynamic response. In this paper such effects are accounted for in closed-form expressions. The evaluation of the expressions requires knowledge of estimates of the variation of the modal exciting force with wave incidence angle. However, only the relative variation of the modal exciting force as a percent of that at an arbitrarily chosen reference angle is required.

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Evaluation of the wave force in absolute terms still is not required.

There are numerous applications of present interest. For example, the fatigue analysis of a tension-leg platform must include an estimate of the amplified responses at the natural frequencies of the structure in heave, pitch, and roll. This method quickly provides that response estimate. An example calculation for the heave response of a tension-leg platform is included. Two additional examples are provided, which exploit simplifications that frequently may be useful. The first is the case where the wave exciting force is independent of incidence angle, as would be true when considering the heave response of a structure with a vertical axis of symmetry. The second example illustrates the simplifications obtained when the wave spectrum is distributed broadly in incidence angle.

The techniques applied in this paper are new to the field of ocean engineering. However, they are not without precedent and have found extensive application in the fields of acoustics and vibration.[3]

LINEAR OSCILLATOR MODEL

A structure in the ocean may have a large number of natural frequencies, although at only a few is the dynamic response to wave excitation likely to be important. It is convenient for the purpose of this paper to assume that by using the techniques of modal analysis each of the responding natural modes may be modeled as an independent single-degree-of-freedom resonator. The general requirements for this are that the vibration of the structure behave in a linear fashion and that the damping be small. The motivation for using modal analysis is that it is far simpler mathematically to analyze a few independent single-degree-of-freedom models than one large, coupled, multidegree-of-freedom system. Reference 4 presents a thorough discussion of modal analysis, and Reference 5 demonstrates its application to offshore structures. In some cases the technique of modal analysis does not eliminate all of the damping-related coupling terms between modes. For the response predictions considered in this paper this generally is not a problem. Supporting discussion is presented later.

This paper will be presented in terms of the response of a simple single-degree-of-freedom resonator excited by ocean wave forces. The results should be interpreted in the larger context of modal analysis: that the total response of a structure can be obtained by a superposition of the individual responses of the modes of interest. Although it will not always be stated explicitly, the coefficients and variables of the single-degree-of-freedom system must be expressed in terms of the appropriate modal quantities for the specific natural mode being modeled.

The equation of motion for the single-degree-of-freedom resonator excited by ocean waves will contain terms corresponding to hydrodynamic forces as well as purely mechanical ones, such as structural stiffness. The hydrodynamic exciting forces usually will be a function of the relative acceleration, velocity, and displacement between the water particles and the generalized coordinates that represent the motion of the structure. For structures that behave in a linear fashion, these quantities may be expressed separately. Thus, the loads on the resonator resulting from its motion in an otherwise calm ocean may be added to the forces exerted on the resonator when held rigidly in place and loaded by the passage of ocean waves. This may be expressed mathematically as follows, where the coefficients are often functions of frequency.

$$(m+m_a)\ddot{s}+(R_i+R_{rad}+R_v)\dot{s}+(K_s+K_{hy})s = f(\ddot{\eta})+g(\dot{\eta})+h(\eta), \quad (1)$$

where the response quantities are:

- m = modal mass of structure,
- m_a = modal added mass of water,
- R_i = linear internal structural modal damping, not related to the presence of the fluid,
- R_{rad} = radiation or wave-making damping of the mode (a linear frequency dependent term that may be expressed by potential flow theory),
- R_v = viscous fluid modal damping (due to the assumption of light damping, it is assumed that an equivalent linearization will be adequate),
- s = appropriate normal coordinate obtained by modal analysis for this particular mode,
- K_s = structural modal stiffness, and
- K_{hy} = hydrostatic modal stiffness that arises from changes in displacement of a body on the free surface.

On the right-hand side appear the excitation quantities that are functions of the water particle acceleration, velocity, and displacement $\ddot{\eta}$, $\dot{\eta}$, and η, respectively.

- $g(\dot{\eta})$ = drag force excitation term that is assumed small compared with the other two terms and is dropped, and
- $f(\ddot{\eta}), h(\eta)$ = hydrodynamic modal forces that normally would be calculated from potential flow theory by integrating the pressure over the surface of the body; these are, in fact, the inertial and hydrostatic forces exerted by passing waves.

The exciting forces appearing on the right-hand side are the modal forces that would be exerted on the body if it were held rigidly in place. These forces include all linear diffraction effects. A principal conclusion of this paper is that these forces need not be evaluated explicitly to obtain an estimate of the mean square response of a particular vibration mode.

For the assumption of wave force linearity to be valid, the ratio of wave amplitude to structural member diameter must be on the order of one or less. For circular members in oscillating flow this corresponds to a Keulegan Carpenter number of less than 2π. The results presented in this paper are not valid for structures composed of small members exposed to large waves. However, high-cycle low-stress fatigue considerations require estimates of response in low sea states, where for even relatively small members the forces are essentially linear. In such circumstances the results presented here will be useful.

Equation 1 is of the form of a simple single-degree-of-freedom oscillator, as simplified by

$$m_{tv}\ddot{s} + R_T\,\dot{s} + Ks = F(t), \qquad (2)$$

where:

m_{tv} = total virtual mass,
R_T = total damping,
K = total stiffness, and
$F(t)$ = modal exciting force.

The undamped natural frequency and the damping ratio are given by these familiar expressions:

$$\omega_o = \sqrt{K/m_{tv}}, \qquad (3)$$

$$\zeta = \frac{R_T}{2\omega_o m_{tv}} \qquad (4)$$

m_{tv}, R_T, and K generally may not be assumed independent of frequency. However, in the following analysis, the frequency range of interest is confined to a narrow band about the natural frequency. Within this band we assume that m_{tv} and K do not vary. However, the frequency dependence of R_T may not be disregarded so easily. The radiation damping portion of R_T is strongly frequency dependent. Because the behavior of an oscillator at resonance is damping controlled, the nature of the damping must be well-understood before simplifying assumptions are made.

RECIPROCITY RELATIONS

The evaluation of hydrodynamic forces on a body in an incident wave system is difficult. It is necessary to know not only the hydrodynamic pressure in the incident wave system but also the effects on this pressure field due to the presence of the body. The incident pressure field is relatively easy to evaluate, but the diffraction effects usually are extremely difficult to obtain. Haskind[1] and Newman[2] have presented expressions for the exciting forces and moments on a fixed body that do not require knowledge of the diffraction effects but depend instead on the velocity potential for forced oscillations of the body in calm water. In other words, there is a direct relationship between the radiation damping on a body that is forced to oscillate in calm water and the force exerted on that body when it is held fixed in incident waves.

Newman evaluated the expressions for an arbitrary three-dimensional body either on the surface or submerged in terms of the six generalized coordinates and forces relating to the six rigid-body degrees of freedom.

In general, one would desire the relation between the modal radiation damping coefficient and the modal exciting force. The modal exciting force and, therefore, the modal radiation damping may be obtained by a linear transformation from the six generalized forces in accordance with the method of modal analysis.

The Haskind/Newman relation is stated here in terms of the modal quantities necessary in the remainder of this discussion:

$$R_{rad}(\omega) = \frac{\omega^3}{4\pi\rho g^3}\int_0^{2\pi}\frac{|F(\omega,\beta)|^2}{|A(\omega,\beta)|^2}\,d\beta, \qquad (5)$$

where:

$R_{rad}(\omega)$ = radiation damping coefficient for the natural mode of interest,
$F(\omega,\beta)$ = modal exicting force exerted on the fixed body by a system of plane deepwater waves of frequency ω and amplitude $A(\omega,\beta)$, incident on the body at an angle β; $F(\omega,\beta)$ and $A(\omega,\beta)$ both have an $e^{i\omega t}$ time-dependent term which will not be explicitly written out,
ρ = density of water, and
g = acceleration of gravity.

Equation 5 states that the modal radiation damping coefficient is proportional to the integral of the square of the modal exciting force, integrated over all angles of incidence.

Vugts[6] experimentally confirmed the validity of these results in a series of model test published in 1968.

In general, for an arbitrary body the wave forces will depend on the shape of the body and the angle of incidence of the waves. For this analysis it is useful to have a shape function defined as

$$\Gamma(\omega,\beta) = \frac{F(\omega,\beta)}{A(\omega,\beta)} \qquad (6)$$

Γ is a measure of the modal force per unit wave amplitude as a function of wave frequency and incidence angle. A mean square value of Γ computed over all incidence angles is given simply by

$$\langle|\Gamma|^2\rangle_\beta = \frac{1}{2\pi}\int_0^{2\pi}\frac{|F(\omega,\beta)|^2}{|A(\omega,\beta)|^2}\,d\beta \qquad (7)$$

Therefore, from Equation 5, $R_{rad}(\omega)$ may be expressed in terms of the mean square value of :

$$R_{rad}(\omega) = \frac{\omega^3}{2\rho g^3} \langle |\Gamma|^2 \rangle_\beta . \ldots\ldots\ldots\ldots\ldots (8)$$

Equation 6 may be rewritten as

$$F(\omega,\beta) = A(\omega,\beta)\,\Gamma(\omega,\beta). \ldots\ldots\ldots\ldots\ldots (9)$$

This is the modal wave force due to the incidence of regular waves of a single frequency and incidence angle. Again, the time-dependent $e^{i\omega t}$ term is implied and not explicitly written. Because only linear processes are being considered, superposition of waves of many frequencies and incidence angles results in a modal wave force spectrum of this form:

$$S_F(\omega,\beta) = S_\eta(\omega,\beta)\,|\Gamma(\omega,\beta)|^2. \ldots\ldots\ldots (10)$$

When possible the modal force spectrum may be simplified further by integrating this expression over all incidence angles:

$$S_F(\omega) = \int_0^{2\pi} S_\eta(\omega,\beta)\,|\Gamma(\omega,\beta)|^2 d\beta . \ldots\ldots (11)$$

It is desirable to normalize this expression with respect to the simple wave amplitude spectrum and the mean square value of the shape function. The resulting nondimensional normalized modal force is designated by the symbol C_1, which for any given structure, sea state, and natural frequency is a constant expressed as:

$$C_1 = \frac{S_F(\omega)}{S_\eta(\omega)\langle|\Gamma|^2\rangle_\beta} = \frac{\int_0^{2\pi} S_\eta(\omega,\beta)\,|\Gamma(\omega,\beta)|^2 d\beta}{S_\eta(\omega)\langle|\Gamma|^2\rangle_\beta} . \ldots\ldots\ldots (12)$$

Using C_1, the modal wave force spectrum may be expressed as

$$S_F(\omega) = C_1\, S_\eta(\omega)\langle|\Gamma^2|\rangle_\beta . \ldots\ldots\ldots\ldots (13)$$

C_1 is a measure of the influence of directional spreading of the seas of angular dependence of the shape function. As will be shown, $C_1 = 1$ whenever the seas are distributed broadly in direction or the modal force is insensitive to changes in incidence angle. Three examples at the end of the paper show how to obtain C_1.

From Equation 8 the mean square value of Γ may be expressed in terms of the radiation damping. Substitution into Equation 13 results in

$$S_F(\omega) = C_1\, S_\eta(\omega)\,\frac{2\rho g^3}{\omega^3}\, R_{rad}(\omega) . \ldots\ldots (14)$$

This is a result of considerable utility. The wave force spectrum has been expressed in terms of the simple wave amplitude spectrum and the radiation damping. This result leads to useful expressions for the response of the resonator.

RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM RESONATOR TO RANDOM EXCITATION

Through the use of modal analysis, the total structural vibration has been expressed in terms of a set of independent single-degree-of-freedom oscillators, one for each vibration mode. If the displacement of one of these oscillators id denoted by s, the displacement response spectrum to the modal wave force spectrum $S_F(\omega)$ is given by

$$S_s(\omega) = S_F(\omega)\,|H_s(\omega)|^2, \ldots\ldots\ldots\ldots\ldots (15)$$

where $H_s(\omega)$ is the complex frequency response of the resonator and may be found in any vibrations text.[4]

$$|H_s(\omega)|^2 = \left[\frac{1/K^2}{\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_o}\right)^2}\right] . \ldots (16)$$

The modal wave force spectrum and, consequently, the modal radiation damping, $R_{rad}(\omega)$, vary with frequency. The total damping ratio ζ, through its dependence on $R_{rad}(\omega)$, also is a frequency dependent term. The remainder of this section is devoted to presenting a simple but accurate expression for the response of the resonator that arises from the damping-controlled resonant peak that is centered on the natural frequency.

From random vibration theory, the mean square of a process is given by the integral of the spectrum over all frequencies. Therefore, the mean square displacement is given by

$$\langle s^2 \rangle = \int_0^\infty S_s(\omega)d\omega = \int_0^\infty S_F(\omega)\,|H_s(\omega)|^2 d\omega , \ldots\ldots\ldots\ldots (17)$$

where, for engineering purposes, only positive frequencies are allowed.

If the force spectrum is a constant, S_o, over all frequencies, the mean square displacement is simply

$$\langle s^2 \rangle = S_o \int_0^\infty |H_s(\omega)|^2 d\omega . \ldots\ldots\ldots\ldots (18)$$

For light constant damping (i.e., $\zeta \le 0.15$) the value of this integral is approximated closely by this expression, which may be found in the text by Lyon:[3]

$$\langle s^2 \rangle = \frac{\pi S_o}{4 m_{tv}^2 \omega_o^3 \zeta} = \frac{\pi S_o}{2 R_T m_{tv} \omega_o^2} . \ldots\ldots (19)$$

where $R_T = R_i + R_v + R_{rad}$, the total damping of the resonator. The largest contribution to this integral comes from the damping-controlled peak in $|H_s(\omega)|^2$, which is confined to a narrow band of frequencies about the natural frequency ω_o. In fact, 64% can be attributed to the small band in frequency, $\omega_o \pm \zeta\omega_o$, known as the half-power bandwidth, $\Delta\omega = 2\zeta\omega_o$. The mean square response to S_o in the half-power band may be expressed as

$$\langle s^2 \rangle_{\Delta\omega} = S_o \int_{\omega_o(1-\zeta)}^{\omega_o(1+\zeta)} |H_s(\omega)|^2 d\omega$$

$$\simeq \frac{S_o}{R_T m_{tv} \omega_o^2} \text{, } \ldots\ldots\ldots\ldots\ldots (20)$$

$$\therefore \frac{\langle s^2 \rangle_{\Delta\omega}}{\langle s^2 \rangle} = \frac{2}{\pi} = 64\% \text{ . } \ldots\ldots\ldots\ldots (21)$$

If the limits of integration in Equation 20 are doubled to include two half-power bandwidths, $\omega_o \pm 2\zeta\omega_o$, then 80% of the total dynamic response will be included:

$$\langle s^2 \rangle_{2\Delta\omega} \simeq \frac{0.4\pi S_o}{R_T m_{tv} \omega_o^2} \text{ . } \ldots\ldots\ldots\ldots\ldots (22)$$

An accurate estimate of the mean square response of a lightly damped resonator excited by ocean waves may be obtained in a half-power bandwidth. This may be done by assuming that the values of the wave force spectrum and the radiation damping at the natural frequency of the resonator, ω_o, represent acceptable averages over the band $\Delta\omega$. This assumption provides a simple but reasonably accurate estimate of the damping-controlled dynamic response in the half-power band, $\Delta\omega$:

$$\langle s^2 \rangle_{\Delta\omega} \simeq \frac{S_F(\omega_o)}{R_T(\omega_o) m_{tv} \omega_o^2} \text{ . } \ldots\ldots\ldots\ldots (23)$$

The error introduced by this approximation is related directly to the width of the half-power band $\Delta\omega = 2\zeta\omega_o$ and, therefore, to the total damping ζ. For very low damping ($\zeta \leq 0.05$) the error is negligible. This was confirmed by a numerical integration of Equation 20 over the half-power band for a variety of cases in which the wave force spectrum and radiation damping were allowed to vary with frequency in a realistic fashion. The worst case results indicate that the error introduced by using the approximation of Equation 22 was less than 2% for $\zeta = 0.05$. This error will increase with an increase in the tota damping ζ. However, for any specific application the frequency dependence of the wave force spectrum $S_F(\omega)$ and the total damping ratio ζ may be estimated in the neighborhood of the natural frequency ω_o. The actual error may be accounted for by evaluating the ratio between the expressions provided in Equations 23 and 20. Such a procedure would allow the extension of the simple results of Equation 20 to include total damping values as high as 10 or 15%.

In the case of very low total damping ($\zeta \leq 0.05$) the assumption of constant force spectrum and total damping may be increased to include a greater portion of the damping-controlled peak. For example, Equation 22 may be used to provide an estimate of the damping-controlled response in a region which is two half-power bandwidths wide. For $\zeta = 0.05$ the worst case error increases to only 6%, and approximately 80% of the total dynamic response is contained in the prediction given by

$$\langle s^2 \rangle_{2\Delta\omega} \simeq \frac{0.4\pi\, S_F(\omega_o)}{R_T(\omega_o) m_{tv} \omega_o^2} \text{ . } \ldots\ldots\ldots\ldots (24)$$

To simplify the presentation in the remainder of the paper, response estimates will be made for the region defined by a single half-power bandwidth using Equation 23. It is implied that other estimates using broader bands, such as Equation 24, also may be used, though larger errors will results.

ELIMINATION OF EXPLICIT CALCULATION OF WAVE FORCES

The reciprocity relation was used to derive an expression for the modal wave force spectrum in terms of the radiation damping (equation 14). This expression may be substituted in Equation 23 to obtain an expression for the mean square response in the half-power band, which does not require explicit calculation of the wave force spectrum:

$$\langle s^2 \rangle_{\Delta\omega} = \frac{2C_1 \rho g^3 S_\eta(\omega_o)}{m_{tv} \omega_o^5} \frac{R_{rad}(\omega_o)}{R_T(\omega_o)} \text{ . } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The most important feature revealed by this expression is that the damping-controlled response of a resonator excited by linear ocean wave forces is dependent on the ratio of the radiation to total damping evaluated at the natural frequency, ω_o. It is often easier to estimate the ratio $R_{rad}(\omega_o)/R_T(\omega_o)$ than it is to evaluate $R_{rad}(\omega_o)$. Furthermore, because this ratio can never exceed one, an upper bound estimate still may be achieved without any knowledge of the ratio. This upper bound is independent of damping. The widely held belief that the response of a structure at a natural frequency increases without bound as the damping is decreased is simply not true when the excitation is provided by linear wave forces. This is a consequence of the reciprocity relation stated in Equation 5. It is impossible to reduce the radiation damping without also reducing the exciting forces, thus resulting in a bounded response.

The unevaluated constant C_1 is dependent on the shape of the structure and the directionality of the wave spectrum. The following three examples will evaluate C_1. These examples were selected because they may be extended directly to a large variety of ocean structures.

SAMPLE RESPONSE CALCULATIONS

Example 1: Heave Response of an Oceanographic Mooring

The results of this example apply to any structure for which it may be argued that the modal force is independent of wave incidence angel

Consider the simple oceanographic mooring shown in Figure 1. It consists of a submerged spherical float and a tripod elastic tether. The undamped natural frequency in heave is given by Equation 3, where K is a linear stiffness coefficient for small vertical motions. The modal force for vibration in the vertical direction is simply the generalized force in the vertical direction on the float. Furthermore, because the float has a vertical axis of symmetry, the heave exciting force is independent of the angle of incidence of the waves; therefore, $\Gamma(\omega,\beta)$ is a function of ω only.

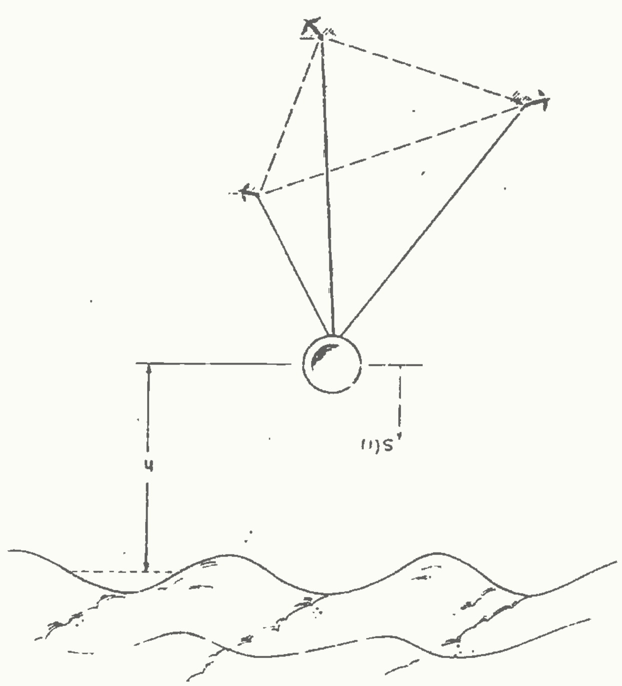

Figure 1. Oceanographic Mooring.

The modal exciting force is derived from Equation 11. Because Γ is independent of β, it may be moved outside of the integral. Note that in this case the magnitude squared of Γ and its means square with respect to β must be equal:

$$|\Gamma|^2 = \langle |\Gamma|^2 \rangle_\beta \quad \ldots\ldots (26)$$

Therefore,

$$S_F(\omega) = \langle |\Gamma|^2 \rangle_\beta \int_0^{2\pi} S_\eta(\omega,\beta)\, d\beta \quad \ldots\ldots (27)$$

$$= \langle |\Gamma|^2 \rangle_\beta S_\eta(\omega), \quad \ldots\ldots (28)$$

because the integration of the directional wave spectrum over all incidence angles results in the simple wave amplitude spectrum.

This result, when substituted into Equation 12, reveals that $C_1 = 1$. It follows from Equation 14 that the heave exciting force spectrum is given by

$$S_F(\omega) = S_\eta(\omega) \frac{2\rho g^3}{\omega^3} R_{rad}(\omega), \quad \ldots\ldots (29)$$

where $R_{rad}(\omega)$ is the modal radiation damping of the axi-symmetric float for heave motions.

The heave response spectrum is as presented in Equation 15, and the mean square response in the small half-power band about the natural frequency is from Equation 25.

$$\langle s^2 \rangle_{\Delta\omega} = \frac{2\rho g^3}{m_{lv}\omega_o^5} S_\eta(\omega_o) \times \frac{R_{rad}(\omega_o)}{R_T(\omega_o)} \,. \quad \ldots\ldots (30)$$

This estimate of the heave response of the buoy is appropriate within the half-power band $\Delta\omega = 2\zeta\omega_o$, provided the system is reasonably linear, the total damping is small, and the assumptions and limitations of modal analysis are satisfied.

In this prediction of the heave response of a mooring, no mention was made of the dependence on the depth of submergence. This is implicit in the ratio $R_{rad}(\omega_o)/R_T(\omega_o)$. Newman shows that the radiation damping coefficient decreases as e^{-2kh}, where k is the wave number of radiated waves. In the limit that the depth of submergence $h \to \infty$, then $R_{rad}(\omega_o) \to 0$, and the ratio also goes to zero. Thus the response of the buoy is predicted correctly to be zero at depths below the region of significant wave excitation.

The specific results shown in Equations 29 and 30 for this example are generally applicable to a broad range of structures—that is, whenever the modal exciting force is independent of wave incidence angle. As shown next, these results also apply whenever the waves in the frequency band of interest can be assumed to have random incidence angle.

Example 2: Random Incidence Waves

When the incident wave spectrum is distributed equally over all incidence angles, the results shown in Equations 29 and 30 apply. This is relatively easy to demonstrate, even for structures with complicated or unkonwn shape functions. For waves of completely random incidence angle, the directional wave spectrum and the simple amplitude spectrum are related in this way:

$$S_\eta(\omega,\beta) = \frac{1}{2\pi} S_\eta(\omega) \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

This may be substituted into Equation 11, the general expression for the force spectrum:

$$S_F(\omega) = \int_0^{2\pi} S_\eta(\omega,\beta)\, |\Gamma(\omega,\beta)|^2 d\beta$$
$$= S_\eta(\omega)\frac{1}{2\pi}\int_0^{2\pi} |\Gamma(\omega,\beta)|^2 d\beta\,, \ldots\ldots (32)$$

where the angular independent wave spectrum has been moved outside of the integral. The integral now is reduced to that which defines the mean square of Γ with respect to β. Therefore, $S_F(\omega) = S_\eta(\omega)\langle|\Gamma|^2\rangle_\beta$. which leads to the conclusion that $C_1 = 1$. This immediately leads to the same expression for the response in the half-power bandwidth as found in Equation 30 of the previous example. In fact, for the result shown in Equation 30 to be valid, it is necessary that only the waves whose frequencies lie within the half-power band be randomly incident. Waves outside of the band need not be so randomly oriented. As a practical matter, the high-frequency components of a seaway tend to be more confused in direction than the low-frequency waves. Therefore, the validity of the assumption of randomly incident waves may be more appropriate than ordinarily supposed, depending on the natural frequency of the structure, geographic location, and prevailing weather.

This result applies to an arbitrary shape function. Any structural symmetries will reduce the range of angles over which the waves must be randomly incident. For example, it can be shown that for a structure with two orthogonal vertical planes of symmetry, such as a steel jacket platform with a rectangular layout of its primary legs, the waves in the half-power band need only be randomly incident over a semicircle (i.e., 180°) for Equations 28, 29, and 30 to hold. The result might be used to predict the mean square response of the two lowest flexural modes.

For many structures these simplifying assumptions may be justified, and the simple result for the mean square response within the half-power bandwidth as shown in Equation 30 may be applied.

However, at times such assumptions may not be acceptable, and it may be necessary to measure or estimate $\Gamma(\omega,\beta)$ and to incorporate a directional wave spectrum $S_\eta(\omega,\beta)$ Such a procedure is followed in the final example.

Example 3: The Response of a Tension-Leg Platform to Random Wave Excitation

An important concern in contemporary design of all platforms is fatigue. The prediction of the fatigue life is a process that must include the anticipated wave statistics and response statistics of the structure. Numerous authors have reported on difficulties encountered in estimating the response at the resonant frequencies of the structure and have noted that the response prediction for the frequency band about resonance is critically dependent on the value of damping that is selected. This method is directed specifically at predicting the response in the resonant band and puts the role of damping in the proper perspective. Knowledge of the total damping is not sufficient. It is important to know the way in which the damping is distributed among radiation and all other sources.

Consider the hypothetical square tension-leg platform shown in Figure 2. At the preliminary design stage it would be useful to have an estimate of the response of the structure to a prescribed sea state at its natural frequencies in heave, pitch, and roll. In the following example only the heave response will be estimated. The response in the roll and pitch modes would be carried out in a very similar fashion, as has been shown in a thesis supervised by the author.[7] The primary purpose of this example is to illustrate the method one might use to take the geometry of the structure and the directionality of the wave spectrum into consideration.

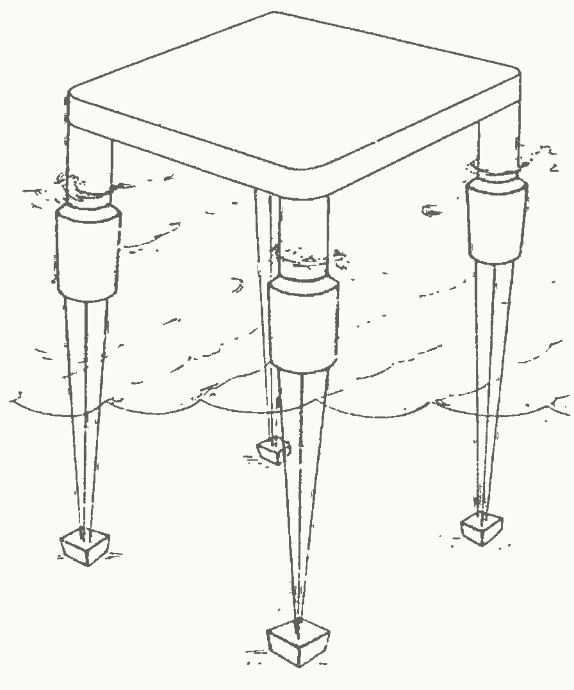

Figure 2. Tension-leg Platform.

The influence of both the directionality of the wave spectrum and the geometry of the structure has been compressed into the unknown constant C_1 shown in Equation 25, the prediction of the mean square displacement response in the half-power bandwidth. C_1 was defined in Equation 12, which is shown here where the integral form of the mean square of $\Gamma(\omega,\beta)$ has been used to replace the $\langle\ \rangle$ notation.

$$C_1 = \frac{\int_0^{2\pi} S_\eta(\omega,\beta)\,|\Gamma(\omega,\beta)|^2 d\beta}{S_\eta(\omega)\,\frac{1}{2\pi}\int_0^{2\pi} |\Gamma(\omega,\beta)|^2 d\beta} \quad \ldots\ldots (33)$$

The directional wave spectrum is prescribed and here is assumed to be a cosine squared distrubution about some reference angle β_o.

$$S_\eta(\omega,\beta) = \frac{2}{\pi} S_\eta(\omega)\cos^2(\beta-\beta_o) \quad \ldots\ldots (34)$$

which is valid for $-\pi/2 \leq \beta - \beta_o \leq \pi/2$ and zero elsewhere. It is noted that

$$S_\eta(\omega) = \int_{\beta_o-\pi/2}^{\beta_o+\pi/2} S_\eta(\omega,\beta)\,d\beta \quad \ldots\ldots\ldots\ldots (35)$$

By substituting into Equation 33 the expression for $S_\eta(\omega,\beta)$ and noting that the common term $S_\eta(\omega)$ cancels out, this is obtained:

$$C_1 = \frac{\int_{\beta_o-\pi/2}^{\beta_o+\pi/2} \frac{2}{\pi}\cos^2(\beta-\beta_o)\,|\Gamma(\omega,\beta)|^2 d\beta}{\frac{1}{2\pi}\int_0^{2\pi} |\Gamma(\omega,\beta)|^2 d\beta} \quad \ldots\ldots (36)$$

The problem has reduced to the need for an estimate of the angular dependence of $|\Gamma(\omega,\beta)|$. This task is simplified because an expresion valid for all frequencies, ω, is not necessary. An estimate valid at only the natural frequency of interest, ω_o, is sufficient. In Figure 3, plane progressive deepwater waves of unit amplitude and frequency, ω_o, are shown approaching the tension-leg platform at an angle β. The magnitude of the heave force exerted on a single axially symmetric leg is independent of incidence angle and may be expressed as $|\Gamma_o(\omega)|$.

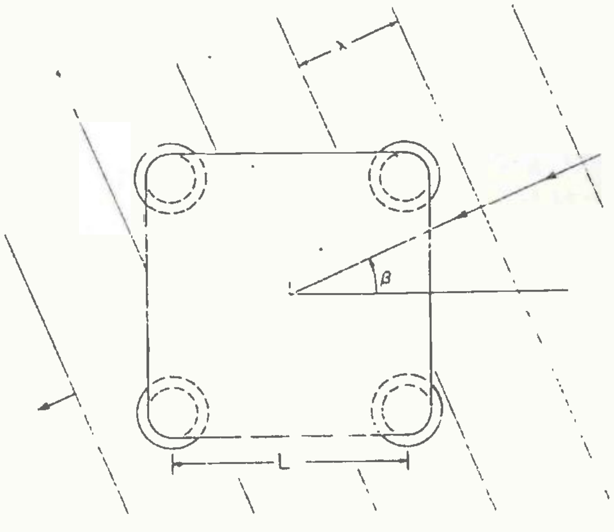

Figure 3. Regular Waves Incident on the Tension-leg Platform.

The magnitude of the force exerted on the entire structure will depend primarily on the relative phases of the four individual leg forces and on any leg interaction effects. The interaction effects are assumed small compared with the phase effects and are ignored. The magnitude of the total heave force accounting for phase effects is given by

$$|\Gamma(\omega_o,\beta)| = 4|\Gamma_o(\omega_o)|\cos\left(\frac{\pi L}{\lambda}\cos\beta\right) \cdot \cos\left(\frac{\pi L}{\lambda}\sin\beta\right), \quad \ldots\ldots (37)$$

where L is the leg spacing and λ is the wave length corresponding to a frequency ω_o. Substitution of this expresion into Equation 36 yields this result for C_1:

$$C_1 = \left[\int_{\beta_o-\pi/2}^{\beta_o+\pi/2} \cos^2(\beta-\beta_o)\cos^2\left(\frac{\pi L}{\lambda}\cos\beta\right) \cdot \cos^2\left(\frac{\pi L}{\lambda}\sin\beta\right) d\beta \Bigg/ \int_0^{2\pi} \cos^2\left(\frac{\pi L}{\lambda}\cos\beta\right) \cdot \cos^2\left(\frac{\pi L}{\lambda}\sin\beta\right) d\beta\right] \quad \ldots\ldots (38)$$

This expression was integrated numerically for all combinations of heave natural period and leg spacing ranging from 1 to 4 seconds and 100 to 300 ft (30.5 to 91.5 m).[7] To 0.1% accuracy, $C_1 = 1$ for all directions of incidence, β_o, of the cosine squared wave spectrum. The cosine squared distribution was sufficiently broad to smooth out the effects of varying wave force phases on the four legs. This unexpected but simple conclusion allows the use of the simple result of the previous two examples. The mean square heave response in the damping-controlled half-power band is given by Equation 30.

For the large legs of a tension-leg platform the radiation damping will likely be the greatest contributor to the total damping. Consequently, a conservative but reasonable upper bound estimate for the ratio of the radiation to total damping is 1, and Equation 30 reduces to

$$\langle s^2 \rangle_{\Delta\omega} = \frac{2\rho g^3 S_\eta(\omega_o)}{m_{tv}\,\omega_o^5} \quad \ldots\ldots (39)$$

An example calculation where:

m_{tv} = 22,000 tons (20,000 Mg), the virtual mass of the tension-leg platform in heave,

ω_o = 2.1 radians/s, which corresponds to a heave period of 3 seconds, and

$S_\eta(\omega_o)$ = 0.204 ft²-s (189 × 10⁻² m²-s), calculated for a 30-knott (15-m/s) Pierson-Moskowitz spectrum,

Yields a root mean square heave amplitude of

$$\sqrt{\langle s^2 \rangle_{\Delta\omega}} = \tfrac{1}{4} \text{ in. (6.7 mm)} \quad \ldots\ldots (40)$$

The heave response is insignificant. However, to arrive at that conclusion by any other means would have been much more difficult.

Damping-Induced Coupling

At this point it is appropriate to discuss a routinely ignored source of error for all response prediction techniques. The error arises because of the coupling between otherwise independent vibration modes that is introduced through damping.

Damping-induced coupling makes it possible for vibration energy to be transferred between modes. For the response prediction analysis presented in this paper, such coupling generally is not significant for the following reasons.

First, for most ocean structures of interest only a few modes have low enough natural frequencies to be excited by the wave spectrum. With a few notable exceptions the natural frequencies of these modes tend to be well separated. Due to the assumption of light total modal damping, the response of each mode is dominated by the damping-controlled peak centered on the natural frequency. As long as no two natural frequencies are so close together that their response peaks overlap, then the energy transfer by damping-induced coupling between any two modes will be insignificant.

In certain types of structures, coincident natural frequencies do occur. Two common examples are the lowest end-on and broadside flexural natural frequencies of steel jacket structures and the pitch and roll natural frequencies of a square tension-leg platform as described in the previous example. In both cases, however, the rectangular or square geometries of the structures provide symmetries in the motion of each mode that results in negligibly small damping-related coupling.

For example, the response of the pitch mode of the tension-leg platform will result in port-starboard symmetry of radiated waves. As a consequence, the radiated waves will generate no roll-exciting moment. Therefore, even though the response peaks overlap, no coupling results from the radiation component of the total damping. Smiliar arguments may be applied to the other damping components. Small asymmetries that do occur in structures give rise to small coupling terms, which often may be neglected.

ENGINEERING IMPLEMENTATION OF THESE RESULTS

The implementation of new theoretical results often requires alteration of accepted engineering practice. The theoretical importance of the ratio of radiation to total damping has not been recognized previously. Experimental techniques and numerical tools for efficient evaluation of the ratio are unavailable. Experience will reveal which applications are best suited to the methods described here. A comparison with present practice is used to highlight promising features of the new techniques.

Present practice in dynamic response prediction requires estimation of the magnitude of the wave amplitude to wave force transfer function as a function of wave frequency and wave incidence angle. This function is denoted by $|\Gamma(\omega,\beta)|$ in this paper. For comparison the method proposed here does not require an absolute measure of $|\Gamma(\omega,\beta)|$ but only its relative variation with respect to that at an arbitrary incidence angle. Thus, it is easier to estimate and less sensitive to selection of, for example, the exact value of the inertia coefficient.

Present practice also requires an independent estimate of the total damping of the structural natural mode of interest. The recent works of Ruhl and Berdahl [8] and Vandiver and Campbell [9] reveal that the published results for measured modal damping on existing structures has been generally inaccurate. Therefore the ability to estimate the total damping for new designs has been hampered by a lack of accurate empirical data.

The method proposed also requires knowledge of the model damping but in the rather unique form of the ratio of the radiation component to total damping. In some cases this may be easier to estimate as it depends only on relative quantities. Furthermore, it is helpful that in the absence of reliable knowledge of the damping, an upper bound on the ratio may be used until further information becomes available.

A number of possibilities exist for the engineering estimation of R_{rad}/R_T and $|\Gamma(\omega,\beta)|$. Consider the estimation of $|\Gamma(\omega,\beta)|$ first. In cases of special symmetry or diffuse seas as in Examples 1 and 2, $|\Gamma(\omega,\beta)|$ need not be estimated at all. For more complex structures and/or directionally concentrated wave spectra, the relative variations of $|\Gamma(\omega,\beta)|$ with β may be estimated by one of the following methods. The first is by carefully considered engineering approximation as illustrated in Example 3. The second is by a static, not dynamic, finite element time domain model of the structure, in which unidirectional regular waves at the natural frequency of interest are passed by the structure from many different incidence angles. Thereby, $|\Gamma(\omega_o,\beta)|$ is obtained for each value of incidence angle. The third method is by a relatively simple static, not dynamic, model test. In such a model test unidirectional regular waves would be passed by the model held rigidly in place by a sufficient number of load cells to determine the modal exciting force. In sequential runs the incidence angle of the model would be varied to develop $|\Gamma(\omega_o,\beta)|$. Such a test requires geometric similarity of the model and Froude scaling, but not dynamic similitude of the mass and stiffness distribution. Estimation of modal exciting forces for rigid body modes such as heave and pitch of a tension-leg platform are extracted easily from load cell data. For structural modes that exhibit deformation of the structure at locations exposed to significant wave forces, estimation of modal exciting forces from load cell data requires additional correction for mode shapes and, therefore, is more difficult.

A number of techniques exist for the estimation of R_{rad}/R_T. For simple structures, such as a freestanding caisson, R_{rad} may be calculated analytically from potential flow theory. For more complex structures such as tension-leg platforms, a diffraction theory wave force program may be used. Coupled with estimates of the remaining sources of damping, the analysis leads to an estimate of R_{rad}/R_T.

Another method for obtaining R_{rad} is by direct application of the principle of reciprocity. Data for $|\Gamma(\omega_o,\beta)|$ may be obtained by model test or finite element simulation as described previously. The mean square of that data with respect to incidence angle, $\langle|\Gamma(\omega_o,\beta)|^2\rangle_\beta$, may be used directly in Equation 8 to obtain an estimate of R_{rad}.

R_{rad}/R_T also may be hindcast from full-scale response data. To do this would require simultaneous measurement of the dynamic response and the directional wave spectrum, plus an independent estimate of the directional dependence of $|\Gamma(\omega,\beta)|$. Given such data, the constant C_1 could be estimated and Equation 25 could be solved for R_{rad}/R_T. This procedure has been attempted and reported for the lowest flexural modes of two separate pile-supported platforms. The first was a small four-leg jacket in 70 ft (21.3 m) of water. The hindcast value of $R_{rad}/R_T = 0.1$.[10] The second estimate was obtained for an eight-leg production platform in 325 ft (99 m) of water. That estimate was $R_{rad}/R_T = 0.08$.[10] Both hindcast estimates required numerous approximations and small data sets; therefore, rather large confidence bounds were implied. Field experiments to yield accurate estimates of R_{rad}/R_T are possible but have not been conducted.

CONCLUSIONS

A method has been presented for predicting the damping-controlled dynamic response of an offshore structure. The method is applicable to a wide variety of structures and depends only on the assumptions of linearity of wave forces and structural response. Furthermore, it requires that the total structural damping be small.

There are three principal conclusions to be drawn. First, the linear wave force spectrum on a structure may be expressed in terms of the radiation damping of the structure. This is a consequence of the principle of reciprocity for ocean wave forces, which has been known for many years but has not been applied to many common ocean engineering problems.

Second, through the use of the above result, a method for estimating the damping-controlled response of a structural natural mode has been presented that does not require explicit calculation of the modal wave force spectrum.

Finally, the role of damping in the estimation of dyna[mic] response is placed in the proper perspective. Li[near] wave forces and modal damping are not independ[ent] quantities. Therefore, it is not the total damping [of a] vibration mode that governs the response to wave [ex]citation but, in fact, the ratio of the radiation to t[otal] damping. Since this ratio has an upper bound of 1, response has an upper bound independent of the ex[act] value of the damping.

NOMENCLATURE

$A(\omega,\beta)$ = plane progressive waves of amplit[ude] A, frequency ω, and inciden[ce] angle β
C_1 = constant dependent on $S_\eta(\omega,\beta)$ a[nd] $\Gamma(\omega,\beta)$
$f(\ddot{\eta})+h(\eta)$ = linear wave forces on fixed body
$F(t)$ = modal wave force on fixed structu[re]
$F(\omega,\beta)$ = modal wave force on fixed structu[re] due to waves $A(\omega,\beta)$
g = acceleration of gravity
$g(\dot{\eta})$ = drag exciting force on fixed body
$H_s(\omega)$ = frequency response of linear secon[d] order single-degree-of-freedo[m] system
K = total modal stiffness
K_s, K_{hy} = structural and hydrostatic contrib[u]tions to the modal stiffness
L = leg spacing on tension-leg platform
m = modal structural mass
m_a = modal added mass
m_{tv} = total modal virtual mass
R_i = linear nonhydrodynamic damping
$R_{rad}, R_{rad}(\omega)$ = linear radiation damping
R_T = total linearized damping
R_v = linearized viscous hydrodynamic damping
s = modal displacement coordinate
$\langle s^2\rangle_{\Delta\omega}$ = mean square displacement in the band $\Delta\omega$
$S_F(\omega,\beta)$ = directional modal wave force spectrum
S_o = constant force spectrum
$S_s(\omega)$ = modal displacement spectrum
$S_\eta(\omega)$ = wave amplitude spectrum
$S_\eta(\omega,\beta)$ = directional wave spectrum
β = wave incidence angle
$\Gamma(\omega,\beta)$ = modal wave force $F(\omega,\beta)$ per unit wave amplitude
$\langle|\Gamma|^2\rangle_\beta$ = mean square of $\Gamma(\omega,\beta)$ with respect to β
$\Delta\omega$ = half-power bandwidth
ζ = total modal damping ratio
$\eta, \dot{\eta}, \ddot{\eta}$ = water particle displacement, velocity, and acceleration

λ = wave length with frequency ω_o
ρ = density of water
ω = wave frequency
ω_o = natural frequency of the mode
$| \ |$ = denotes magnitude of

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