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St. John's, Newfoundland
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DYNAMIC ANALYSIS OF LONG COMPOSITE CYLINDRICAL SHELLS SUBMERGED IN AN ACOUSTIC MEDIUM

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ABSTRACT

A theoretical method for analyzing the free and forced vibrational behavior of long composite cylindrical shells in an acoustic medium is presented. The classical laminate theory is used to derive the laminate properties and formulate the equations of motion. The natural frequencies and the mode shapes of vibrations for the cylindrical shells in a vacuum are obtained. The dynamic forces exerted by the surrounding infinite acoustic medium are derived in terms of a series of the vibrational mode shapes. Modal analysis techniques are then used to decouple the equations of motion. Finally, the dynamic responses of the cylindrical shells in an acoustic medium are investigated.

1. INTRODUCTION

The importance of composite materials in hydrospace and aerospace applications is rapidly increasing. The cylindrical shell configuration is widely used in the area of marine and aerodynamic structures. Thus, the dynamic response of the composite cylindrical shells is of great interest to naval architects and aerodynamicists.

The laminate properties of composite cylindrical shells usually are derived from the classical laminate theory. This method can be readily applied to several types of shell construction, including beam stiffened, corrugated and sandwich shells [1]. Isotropic cylindrical shells with closely spaced ring and/or stringer stiffeners can be approximated as specially orthotropic cylindrical shells, which represents one kind of composite cylindrical shell [2,3]. Dong analyzed the free vibration of laminated specially orthotropic cylindrical shells under an arbitrary set of homogeneous boundary conditions [4].

For marine applications, it is important to account for the fluid medium when studying the dynamic characteristics of elastic structures. These characteristics may differ considerably from those in vacuo. The dynamic analysis of isotropic long cylindrical shells in an acoustic medium for the three-dimensional case has been studied by Junger [5]. A similar study in which the modal analysis approach has been applied is described by Bleich and Baron [6]. However, in the latter investigation the primary emphasis was placed on the structural response of the shells and second emphasis on the acoustical considerations. More recently, Geers and Felippa used the Doubly Asymptotic Approximations (DAA) method [7] for vibration analysis of submerged structures. Their analysis utilized the finite element method to discretize the structure.

In this paper, the dynamic analysis of a long composite cylindrical shell with or without simply supports at equal support spacing sumerged in an acoustic medium is carried out by the modal analysis as described in [6]. The natural frequencies and mode shapes of the vibrating system are readily obtained from a structural eigen-analysis. This modal approach is appropriate and relatively inexpensive for the special cylindrical geometry and boundary conditions considered in this paper. However, if the structures are more complex or the boundary conditions are more general, numerical methods such as DAA appear to be more appropriate.

2. FORMULATION OF EQUATIONS OF MOTION FOR FREE VIBRATION IN VACUUM

The differential equations of motion are formulated in terms of the three middle surface displacement components u, v, w. The displacements are assumed to be small compared to the shell thickness which makes the problem linear. The line elements normal to the undeformed middle surface remain straight and normal to the deformed middle surface and they undergo no change in length.

2.1 Stress-Strain Relations

From the classical laminate theory, the stress-strain relations in principal material coordinates for a lamina of an orthotropic composite under plane stress are given by:

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\end{pmatrix} \tag{1}$$

where Q_{ij} are reduced stiffness coefficients.

In a more general case of orthotropic lamina with an angle θ between the principal material axis and the geometrical body axis, the stress-strain relations are given as:

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{pmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{pmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{pmatrix} \tag{2}$$

where $\bar{Q}_{ij}$ are transform reduced stiffnesses.

The values of $\bar{Q}_{ij}$ are expressed in terms of Q_{ij} and angle θ [8]. The values of Q_{ij} can be written in terms of the engineering constants as follows:

$$Q_{11} = \frac{E_{11}}{(1-\nu_{12}\nu_{21})};\quad Q_{12} = \frac{\nu_{12}E_{22}}{(1-\nu_{12}\nu_{21})};\quad Q_{22} = \frac{E_{22}}{(1-\nu_{12}\nu_{21})};$$

$$Q_{66} = G_{12} \quad and \quad \frac{\nu_{12}}{E_{11}} = \frac{\nu_{12}}{E_{22}}$$

2.2 Strain and Displacement Relations of Shells

The linear strain and displacement relations can be written as:

$$\epsilon_x = \epsilon_{x0} + z\chi_{x0}\ ;\ \epsilon_y = \epsilon_{y0} + z\chi_{y0}\ ;\ \gamma_{xy} = \epsilon_{z0} + z\chi_{z0} \tag{3}$$

where the z coordinate is measured from points on the middle surface in the direction normal to the middle surface,

x,y is the axial and circumferential coordinate, respectively,

$\epsilon_{x0} = \frac{\partial u}{\partial x}; \epsilon_{y0} = \frac{\partial v}{\partial y} - \frac{w}{R}$ and $\epsilon_{s0} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ are middle surface strains,

$\chi_{x0} = -\frac{\partial^2 w}{\partial x^2}; \chi_{y0} = -\frac{\partial^2 w}{\partial y^2}$ and $\chi_{s0} = -2\frac{\partial^2 w}{\partial x \partial y}$ are the middle surface curvatures and

u, v, w are the displacements at the middle surface.

2.3 Force, Moment Resultants and the Equations of Motion

The force and moment resultants for n layered laminate are defined as:

$$(N_x, N_y, N_{xy}, M_x, M_y, M_{xy}) = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\sigma_x^k, \sigma_y^k, \tau_{xy}^k, z\sigma_x^k, z\sigma_y^k, z\tau_{xy}^k] dz \tag{4}$$

Using Eqs.(1),(3) and Eq.(4), integrating through the thickness, and substituting in the relevant equilibrium equations, we obtain the equation of motion in terms of the displacements as:

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2} + A_{12}\left(\frac{\partial^2 v}{\partial x \partial y} - \frac{1}{R}\frac{\partial w}{\partial x}\right) + A_{66}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = m_a \ddot{u} \\
&A_{12}\frac{\partial^2 u}{\partial x \partial y} + A_{22}\left(\frac{\partial^2 v}{\partial y^2} + \frac{1}{R}\frac{\partial w}{\partial y}\right) + A_{66}\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2}\right) = m_a \ddot{v} \\
&-D_{11}\frac{\partial^4 w}{\partial x^4} - (2D_{12} + 4D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w}{\partial y^4} \\
&-\frac{1}{R}\left(A_{12}\frac{\partial u}{\partial x} + A_{22}\left(\frac{\partial v}{\partial y} - \frac{w}{R}\right)\right) = m_a \ddot{w}
\end{aligned} \tag{5}$$

where $A_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}^k (h_k - h_{k-1})$ are extensional stiffnesses,

$B_{ij} = 1/2 \sum_{k=1}^{n} \bar{Q}_{ij}^k (h_k^2 - h_{k-1}^2)$ are coupling stiffnesses,

$D_{ij} = 1/3 \sum_{k=1}^{n} \bar{Q}_{ij}^k (h_k^3 - h_{k-1}^3)$ are bending stiffnesses,

m_a is the mass per unit area of the shell.

$i, j = 1, 2, 6$

For the unsymmetric laminate case, the values of the coupling stiffnesses B_{ij} will be included in the governing equations of motion which are not considered in this paper.

2.4 Free Vibration of Long Composite Cylindrical Shells in Vacuum

The cylindrical shells are assumed to oscillate with a natural frequency ω and the displacement components u, v, w are proportional to the harmonic function in ωt. Consequently the explicit form of the solutions are:

$$\begin{aligned}
u &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} U_{mn} \cos\frac{m\pi x}{l} \cos\frac{ny}{R} e^{i\omega t} \\
v &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin\frac{m\pi x}{l} \sin\frac{ny}{R} e^{i\omega t} \\
w &= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} W_{mn} \sin\frac{m\pi x}{l} \cos\frac{ny}{R} e^{i\omega t}
\end{aligned} \tag{6}$$

where U_{mn}, V_{mn}, W_{mn} are coefficients of the series,

m is the number of the deformation half waves in the x direction,

n is the number of the deformation full waves in the y direction,

R, l are the radius and length of support spacing, respectively.

It is readily demonstrated that the simply supported boundary condition is satisfied by Eq.(6). This solution can also be used for long cylindrical shells with no support by substituting the length of the longitudinal half wave L in place l/m.

By substituting Eq.(6) into Eq.(5), we obtain the matrix form of equations of motion as:

$$\begin{bmatrix} T_{11} - \lambda & -T_{12} & T_{13} \\ -T_{12} & T_{22} - \lambda & -T_{23} \\ T_{31} & -T_{32} & T_{33} - \lambda \end{bmatrix} \begin{pmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{pmatrix} = 0 \tag{7}$$

where

$$\begin{aligned}
\lambda &= m_a \omega^2, \\
T_{11} &= A_{11}\left(\frac{m\pi}{l}\right)^2 + A_{66}\left(\frac{n}{R}\right)^2, \\
T_{12} &= A_{12}\left(\frac{m\pi}{l}\right)\left(\frac{n}{R}\right) + A_{66}\left(\frac{m\pi}{l}\right)\left(\frac{n}{R}\right) = T_{21}, \\
T_{13} &= \frac{A_{12}}{R}\left(\frac{m\pi}{l}\right) = T_{31}, \\
T_{22} &= A_{22}\left(\frac{n}{R}\right)^2 + A_{66}\left(\frac{m\pi}{l}\right)^2, \\
T_{23} &= \frac{A_{22}}{R}\left(\frac{n}{R}\right) = T_{32}, \\
T_{33} &= D_{11}\left(\frac{m\pi}{l}\right)^4 + (2D_{12} + 4D_{66})\left(\frac{m\pi}{l}\right)^2\left(\frac{n}{R}\right)^2 + \frac{A_{22}}{R^2} + D_{22}\left(\frac{n}{R}\right)^4.
\end{aligned}$$

To obtain the nontrivial solution in Eq.(7), the determinant of the square matrix must be zero which leads to:

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0 \tag{8}$$

where

$$\begin{aligned}
C_1 &= -(T_{33} + T_{22} + T_{11}), \\
C_2 &= -(T_{13}^2 + T_{23}^2 + T_{12}^2 - T_{22}T_{33} - T_{11}T_{33} - T_{11}T_{22}), \\
C_3 &= -(T_{11}T_{22}T_{33} + 2T_{12}T_{23}T_{31} - T_{13}^2T_{22} - T_{23}^2T_{11} - T_{12}^2T_{33}).
\end{aligned}$$

The three natural frequencies can be obtained from Eq.(8) for each m, n and the mode shapes of corresponding three natural frequencies are obtained through the eigen function solution of Eq.(7).

In the case of mode $n = 0$, the displacement v becomes zero and therefore only two natural frequencies are obtained.

3. DYNAMIC ANALYSIS OF LONG COMPOSITE CYLINDRICAL SHELLS IN AN ACOUSTIC MEDIUM

3.1 Equations of Motion in Terms of Generalized Coordinates

The displacements corresponding to 3 natural frequencies and 3 eigenmodes for each m, n can be written as:

$$\begin{aligned}
u &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=1}^{3} U_{mnk} \cos\frac{m\pi x}{l} \cos n\theta \\
v &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{3} V_{mnk} \sin\frac{m\pi x}{l} \sin n\theta \\
w &= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{k=1}^{3} W_{mnk} \cos n\theta \sin\frac{m\pi x}{l}
\end{aligned} \tag{9}$$

where $\vartheta = \frac{1}{k}$, $k = 1,2,3$ for each modes m and n. In order to simplify the following derivations, all summations and the subscripts m, n will be dropped.

The eigenmodes contain arbitrary factors U_k, V_k, and W_k which are the function of time, and only the ratio of these factors can be determined. By selecting the coefficient W_k as generalized coordinates q_k and using the abbreviations:

$$\phi_{u,k} = \frac{U_k}{W_k}\cos n\theta \cos\frac{m\pi z}{l}$$
$$\phi_{v,k} = \frac{V_k}{W_k}\sin n\theta \sin\frac{m\pi z}{l} \quad (10)$$
$$\phi_w = \cos n\theta \sin\frac{m\pi z}{l}.$$

the displacements u, v and w can be expressed as the functions of three generalized coordinates as:

$$u = q_1\phi_{u,1} + q_2\phi_{u,2} + q_3\phi_{u,3}$$
$$v = q_1\phi_{v,1} + q_2\phi_{v,2} + q_3\phi_{v,3} \quad (11)$$
$$w = (q_1 + q_2 + q_3)\phi_w$$

By exploiting the orthogonality condition associated with the generalized coordinates, the equations of motion can be decoupled for each m, n and k as:

$$M_k\ddot{q}_k + M_k\omega_k^2 q_k = Q_k \quad (12)$$

where the generalized masses $M_k = m_s \int\int_A(\phi_{u,k}^2 + \phi_{v,k}^2 + \phi_w^2)dA$,
A is the surface area of the shells,
m_s is the constant mass per unit area of the shells.

3.2 Free Vibration Analysis in an Acoustic Medium

When the cylindrical shell is submerged in an acoustic medium, the generalized forces Q_k are only due to the radial pressure P_a of the fluid medium on the cylinder of radius R. Eq.(12) can be written in the form:

$$\ddot{q}_k + \omega_k^2 q_k = \frac{\int\int_A P_a\phi_w dA}{M_k} \quad (13)$$

The potential of an acoustic medium is defined by the wave equation :

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \quad (14)$$

where c is the sound velocity in an acoustic medium,
Φ is the velocity potential in an acoustic medium.

The solution of the steady state responses of Eq.(14) for each mode m, n can be written as:

$$\Phi = A_{mn}F(r)\cos n\theta \sin\frac{m\pi z}{l}e^{i\Omega t} \quad (15)$$

where A_{mn} are arbitrary constants,
Ω is the frequency of vibration in an acoustic medium.

Substituting Eq.(15) into Eq.(14), we obtain:

$$F''(r) + \frac{1}{r}F'(r) + (\beta^2 - (\frac{n}{r})^2)F(r) = 0 \quad (16)$$

where $\beta = \sqrt{(\frac{\Omega}{c})^2 - (\frac{m\pi}{l})^2}$

The solution of Eq.(18) which satisfies the condition that $F(r)$ should remain finite as $r \rightarrow \infty$ is:

$$F(r) = \begin{cases} H_n^{(2)}(\beta r), & \text{if } \frac{\Omega}{c} > \frac{m\pi}{l}; \\ H_n^{(1)}(i\beta r), & \text{if } \frac{\Omega}{c} < \frac{m\pi}{l}; \\ \frac{1}{r^n}, & \text{if } \frac{\Omega}{c} = \frac{m\pi}{l}. \end{cases} \quad (1$$

The matching condition at the surface of the shells i that the radial velocity of the displacement, $\dot{w}$, must b equal to the velocity of the particles of fluid medium. Th can be expressed as:

$$\dot{w} = -\frac{\partial\Phi}{\partial r} \quad at \quad r = R \quad (1$$

By substituting Eq.(11) and Eq.(15) into Eq.(18), w will obtain the value of A_{mn} and the velocity potential becomes:

$$\Phi = \frac{-(\dot{q}_1 + \dot{q}_2 + \dot{q}_3)F(r)\phi_w e^{i\Omega t}}{F'(R)} \quad (1$$

where $F(r)$ can be defined for three cases as in Eq.(17)

The radial pressure exerted on the shells by surrounding fluid medium at $r = R$ can be defined as:

$$P_a = -\rho\frac{\partial\Phi}{\partial t} \quad at \quad r = R \quad (20$$

where ρ is the fluid density.

By substituting Eq.(19) into Eq.(20) and introducing

$$\frac{\alpha_k}{m_s} = \frac{\int\int_A \phi_w^2 dA}{M_k} = \frac{W_k^2}{m_s(U_k^2 + V_k^2 + W_k^2)}, \quad (21$$

the equations of motion can then be rewritten as:

$$\ddot{q}_k + \omega_k^2 q_k = \rho(\ddot{q}_1 + \ddot{q}_2 + \ddot{q}_3)\alpha_k\frac{F(R)}{m_s F'(R)} \quad (22$$

For steady state response, we have $q_k = C_k e^{i\Omega t}$ and replacing q_k in Eq.(22), the equations of motion will become

$$(\omega_k^2 - \Omega^2)C_k + \rho\alpha_k\Omega^2\frac{F(R)}{m_s F'(R)}(C_1 + C_2 + C_3) = 0 \quad (23)$$

Expressing Eq.(23) in the matrix form and setting the determinant of the square matrix equal to zero, we obtain the frequency equation as:

$$\frac{\alpha_1}{\omega_1^2 - \Omega^2} + \frac{\alpha_2}{\omega_2^2 - \Omega^2} + \frac{\alpha_3}{\omega_3^2 - \Omega^2} = -\frac{m_s}{\rho\Omega^2\frac{F(R)}{F'(R)}} \quad (24)$$

In the case of $n = 0$, we have only 2 natural frequencies corresponding to 2 mode shapes so that the left side of Eq.(24) will have only 2 terms.

3.3 Forced Vibration Analysis in an Acoustic Medium

An arbitrary radial harmonic force $P(t,\theta,z)$ can be expanded in a Fourier series as:

$$P(t,\theta,z) = \sum_{m=1}^{\infty}\sum_{n=0}^{m} P_{mn}\cos n\theta \sin\frac{m\pi z}{l}e^{i\Omega t} \quad (25)$$

where P_{mn} are arbitrary constants.

The generalized external forces Q_k for each m, n becomes:

$$Q_k = \int_A P_a\phi_w dA - P_{mn}\int_A \phi_w^2 e^{i\Omega t}dA \quad (26)$$

Similar to the case of free vibration, we obtain the equations of motion as:

$$\ddot{q}_k + \omega_k^2 q_k = \rho(\ddot{q}_1 + \ddot{q}_2 + \ddot{q}_3)\alpha_k\frac{F(R)}{m_s F'(R)} - P_{mn}\frac{\alpha_k}{m_s}e^{i\Omega t} \quad (27)$$

The solution of steady state response for Eq.(27) can be written as $q_k = C_k e^{i\Omega t}$. By substituting q_k into Eq.(27), we will obtain 3 non-homogeneous equations which define the value of C_k as:

$$C_k = \frac{\frac{P_{mn}\alpha_k}{\omega_k^2 - \Omega^2}}{m_a(-\rho N \frac{F(R)}{m_a F'(R)} - 1)} \tag{28}$$

where $N = \frac{\alpha_1\Omega^2}{\omega_1^2-\Omega^2} + \frac{\alpha_2\Omega^2}{\omega_2^2-\Omega^2} + \frac{\alpha_3\Omega^2}{\omega_3^2-\Omega^2}$

Substituting C_k into q_k, we will obtain the displacements u, v and w from Eq.(13) as:

$$w = \frac{P_{mn}e^{i\Omega t}\phi_w}{m_a\Omega^2(-\rho\frac{F(R)}{m_a F'(R)} - \frac{1}{N})}$$

$$v = \frac{P_{mn}e^{i\Omega t}\sin n\theta \sin\frac{m\pi x}{l}}{m_a(-\rho N\frac{F(R)}{m_a F'(R)} - 1)}\{\frac{\alpha_1}{\omega_1^2-\Omega^2}\frac{V_1}{W_1} + \frac{\alpha_2}{\omega_2^2-\Omega^2}\frac{V_2}{W_2} + \frac{\alpha_3}{\omega_3^2-\Omega^2}\frac{V_3}{W_3}\} \tag{29}$$

$$u = \frac{P_{mn}e^{i\Omega t}\cos n\theta \cos\frac{m\pi x}{l}}{m_a(-\rho N\frac{F(R)}{m_a F'(R)} - 1)}\{\frac{\alpha_1}{\omega_1^2-\Omega^2}\frac{V_1}{W_1} + \frac{\alpha_2}{\omega^2-\Omega^2}\frac{V_2}{W_2} + \frac{\alpha_3}{\omega_3^2-\Omega^2}\frac{V_3}{W_3}\}$$

The behavior of the radial displacement due to the radial harmonic force will be studied. By letting $\Omega \to 0$, the value of displacement under the static load, w_{st}, will be obtained. The absolute value of $\frac{w}{w_{st}}$ can be derived as:

$$|\frac{w}{w_{st}}| = \frac{1}{\left|(-\rho\Omega^2\frac{F(R)}{m_a F'(R)} - \frac{1}{\frac{\alpha_1}{\omega_1^2-\Omega^2} + \frac{\alpha_2}{\omega_2^2-\Omega^2} + \frac{\alpha_3}{\omega_3^2-\Omega^2}})[\frac{\alpha_1}{\omega_1^2} + \frac{\alpha_2}{\omega_2^2} + \frac{\alpha_3}{\omega_3^2}]\right|} \tag{30}$$

4. NUMERICAL STUDY AND DISCUSSION

The five layered symmetric cross-ply cylindrical shell made from boron epoxy composite is used in this study. The shells have the following material properties:

Major Young's Modulus, 31.0×10^6 psi. Major Poisson's ratio, 0.28

Minor Young's Modulus, 2.7×10^6 psi.

Shear Modulus, 0.75×10^6 psi.

Density, 192×10^{-6} $\frac{lb-sec^2}{in^4}$

The shell geometric properties include: length of support spacing =300 in, radius=90 in, thickness of each layer=.394 in. The isotropic steel shells with the same overall thickness are also considered for comparison purposes. The density of the acoustic fluid is .00112 $\frac{lb-sec^2}{in^4}$ and the sound velocity in the fluid is 59.604 in/sec.

4.1 Free Vibration Study

The frequency equation (Eq.24) is solved graphically for mode m=1,n=1 in Fig.2 for composite shells and in Fig.3 for steel shells. The results suggest similar behavior in the composite shells and steel shells. There is only one real root of frequency Ω. In this mode, the values of the right side of Eq.24 are real until $\Omega = \frac{m\pi c}{l}$,then the values become complex which are not shown in the real plane graphs. The limit value of Ω for which real root was obtained for mode m=1,n=1 is 624 rad/sec. The value of Ω beyond these limit values are associated with the complex roots which represent an outgoing wave which must decay with time[6].

4.2 Forced Vibration Study

The steady state response due to the harmonic external force was also studied. The absolute ratio of $\frac{w}{w_{st}}$ is calculated for the mode as in free vibrations. The results show resonance when the frequency in the fluid Ω, is equal to the real root of the frequency equation. Nondimensionalized displacements for forced vibrations are shown in Fig.4A (m=1,n=1) for composite shells and Fig.5A (m=1,n=1) for steel shells. Beyond the limit values $\Omega = \frac{m\pi c}{l}$ in Fig.4B for composite shells and Fig5B. for steel shells, there is no resonance due to the damping imposed by the complex roots. Another point to note is that there are no responses in some frequencies Ω in the fluid. At such frequencies, the system vibrates like one with a vibration absorber.

5. CONCLUSIONS

The paper demonstrates that the dynamic behavior of composite shells in an acoustic medium resembles that of the steel shells even though the material properties and the stiffnesses of composite material are quite different from steel. However, the dynamic behavior can be quite different if the properties of shell material or fluid are altered. Two extreme cases should be noted:

a. When the stiffness of the shells is very low or the density of the shell material is very high ,the 3 natural frequencies will be lower than the limit value, $\frac{m\pi c}{l}$, and 3 real roots of Ω will be obtained. The cylindrical shells will then vibrate with three resonances in the same manner as they do in a vacuum.

b. When the density of the fluid is sufficiently low, the natural frequencies of the shells will exceed the limit value, $\frac{m\pi c}{l}$, and there will be no real root of Ω at all. In this case, no resonances are expected.

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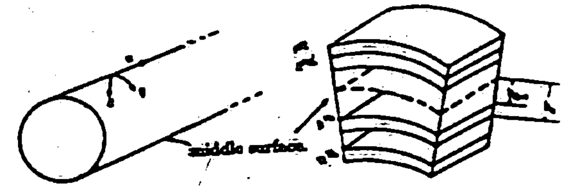

Fig.1 :Composite Cylindrical Shell and Its Cross-section

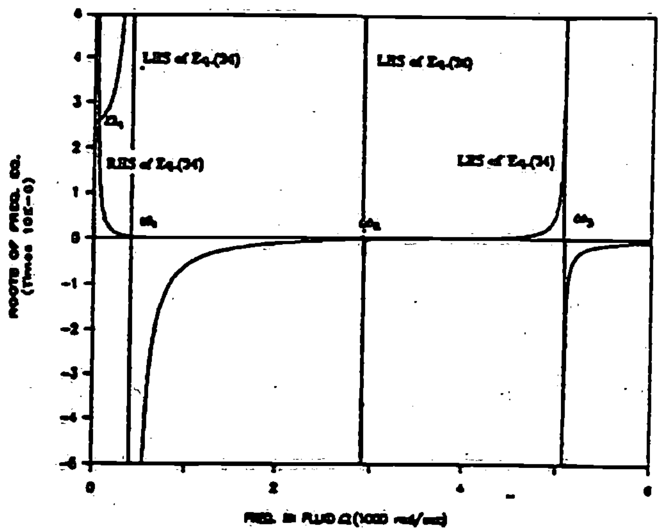

Fig.2 :Roots of Frequency Equation for Free Vibration of 5-Layered Composite(m=1,n=1).

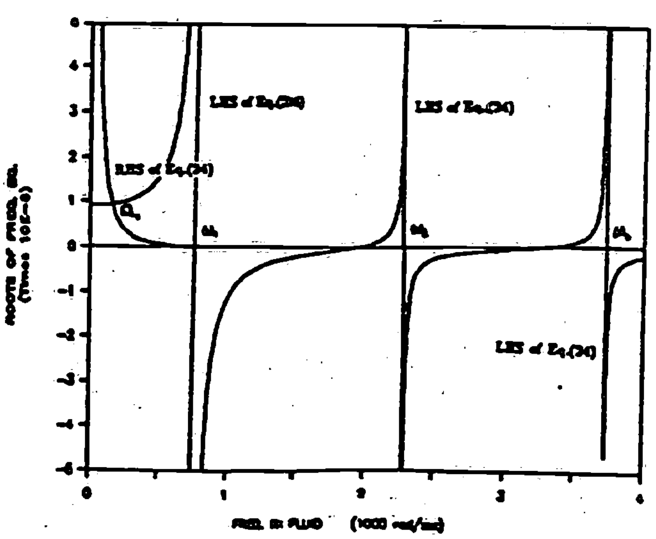

Fig.3 :Roots of Frequency Equation for Free Vibration of a Steel Shell(m=1,n=1).

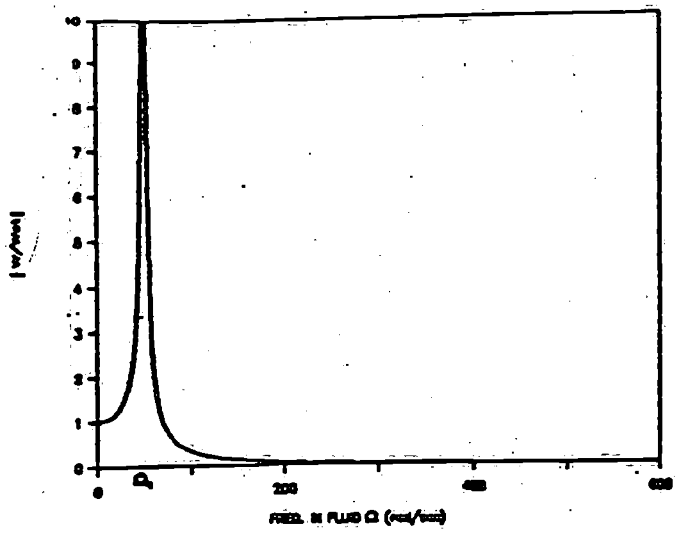

Fig.4A : Ratio of Absolute Radial Displacement for Forced Vibration of a 5-layered Composite Shell(m=1,n=1) for low frequencies.

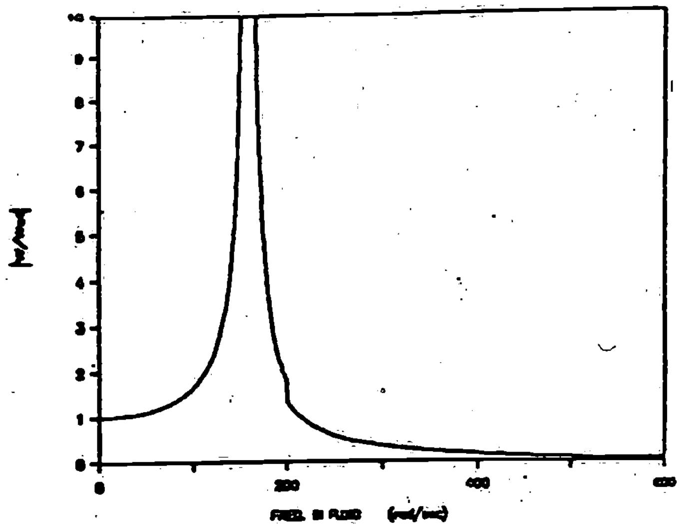

Fig.5A : Ratio of Absolute Radial Displacement for Forced Vibration of a 5-layered a Steel Shell(m=1,n=1) for low frequencies.

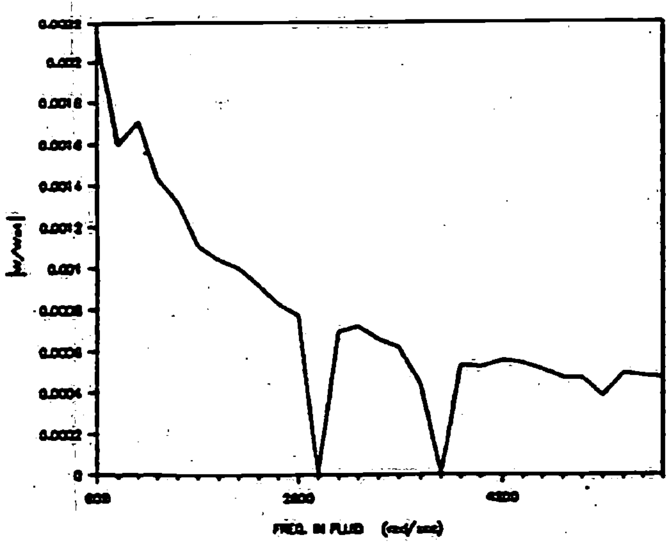

Fig.4B : Ratio of Absolute Radial Displacement for Forced Vibration of a 5-layered Composite Shell(m=1,n=1) for high frequencies.

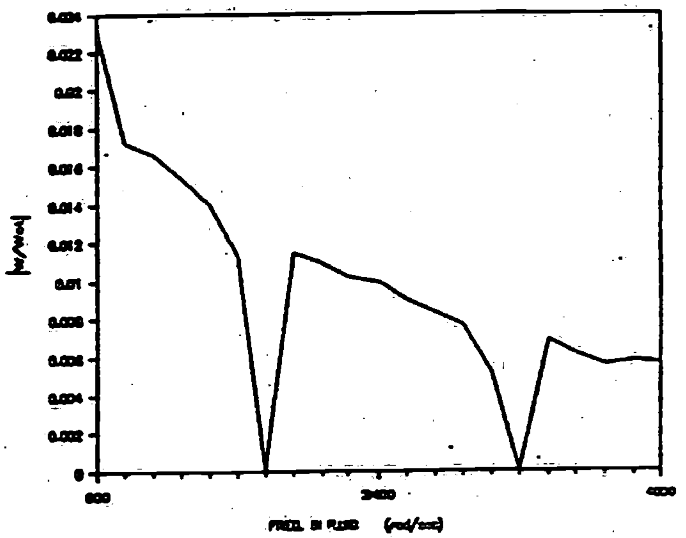

Fig.5B : Ratio of Absolute Radial Displacement for Forced Vibration of a 5-layered a Steel Shell(m=1,n=1) for high frequencies.