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THE INFLUENCE OF SHIP SPEED ON PITCHING, HEAVING, AND HULL DEFLECTION IN WAVES

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Let us consider the pitching and heaving of a ship moving at a constant speed directly into the waves, with relation to a system of plane, regular waves. The coordinates x, y and z are related to the ship (Fig. 1). The fixed system of coordinates ξ, η and ζ coincides at the initial moment of time with the system x, y and z. We will denote up and down movement of the ship in terms of ζ, and its difference, the plus direction of which is shown in Fig. 1, in terms of ψ. The equations for surging by a ship moving at a constant speed in a regular wave pattern, compiled using A.N.Krylov's theory and allowing for the effects of adjoining masses by P.F.Papkovich's method, take the following form:

$$(M + M_1)\ddot{\zeta} + \lambda_\zeta \dot{\zeta} + \gamma F \zeta + S_{M+M_1}\ddot{\psi} + \lambda_{\zeta\psi}\dot{\psi} + S_F \psi = F_0 \cos(\sigma t + \delta_\zeta), \quad (1)$$

$$(K + K_1)\ddot{\psi} + \lambda_\psi \dot{\psi} + \gamma J_y \psi + S_{M+M_1}\ddot{\zeta} + \lambda_{\zeta\psi}\dot{\zeta} + S_F \zeta = M_0 \cos(\sigma t + \delta_\psi);$$

here M and M_1 are the mass of the ship and the connected mass of water;

K and K_1 are the moments of inertia of these masses;

λ_ζ and λ_ψ are the coefficients of damping of the heaving and pitching;

F and $\mathfrak{J}_y$ are the area of the waterline and its moment of inertia relative to the y axis;

S_{M+M_1} and S_F are the static moments relative to the midship section of the mass $M+M_1$ and the area F;

$\lambda_{\xi\psi}$ is the static moment relative to the midship frame of the running coefficient of damping;

F_0, M_0, δ_ξ and δ_ψ are the amplitudes and phase angles of the disturbing force and moment.

The effect of the forward speed of the ship is only taken into account, in equations [1], by taking the frequency of change in the disturbing forces as being equal to the apparent wave frequency,

$$\sigma = \sigma_0 + \frac{2\pi u}{\lambda};$$

here $\sigma_0 = \sqrt{\frac{2\pi g}{\lambda}}$, the wave frequency;

u is the speed of the ship;

λ is the wave length.

When the problem is examined in greater detail we find, however, that the forward speed has an appreciable effect on the magnitude of the terms on the left hand sides of the surging equations, which govern the relationship of the heaving to the pitching.

We will dwell, first of all, on certain experimental results given in reference [1], comparing them with the conclusions which can be drawn when equations (1) are analysed.

Let us consider the equation for heaving, and estimate the order of the terms governing the effects of pitching by comparison with the principal terms. If we estimate the order of the vertical movements by the ship as r_0, and its angular movements as $\frac{2\pi r_0}{\lambda}$, also the distance of the centre of gravity of the waterline area from the midship section as being $x_f \approx 0.015\ L$, we find that the linking terms do not amount to more than 10% of the corresponding principal terms in the equation.

Noting that the resonance pitching and heaving frequencies are almost the same as one another, we reach the conclusion that the interconnection between heaving and pitching, determined using equations (1), is extremely indefinite. Experimental research has also shown that there is a definite relationship between the vertical movements by a ship and its pitching movements. As an example, the results of measuring the movement amplitudes using a model are given in Fig. 2, also calculated data based on using equations (1) separately and together; all the coefficients for the equations were determined by experimenting, with different towing conditions. It follows from Fig. 2 that heaving depends greatly on pitching, and this cannot be explained on the basis of equations (1).

This article contains the necessary changes in the pitching and heaving equations, based on the more accurate determination of the forces acting on a ship moving forwards; the effects of these changes on the heaving and pitching amplitudes, also on the bending moment at the midship section, are estimated.

In reference [2], M.D.Khaskind considered the heaving and pitching problem for a ship, using hydrodynamic methods; he proved that, with a ship moving at a forward speed u = const., an additional vertical hydrodynamic force acts on it (by comparison with the case in which u = 0):

$$\Delta z = \left[-\rho v \iint_S \frac{\partial \varphi_1}{\partial x} \frac{\partial \varphi_1}{\partial n} \, dS - \rho \omega \iint_S \frac{\partial \varphi_2}{\partial x} \frac{\partial \varphi_1}{\partial n} \, dS \right] u e^{i \sigma t} , \qquad (2)$$

where ρ is the mass density of the water;

v, ω are the amplitudes of the vertical and angular speeds of the ship during heaving and pitching;

S is the wetted area of the ship;

φ_1 and φ_2 are the potentials of the induced speeds when the surface S moves with identical speeds in the directions corresponding to heaving and pitching;

n is the direction of the normal to the surface S.

We know that allowing for the force (2) resulted in a substantial change in the conditions for the connection between pitching and heaving. In particular, the equations proved to be linked for a ship symmetrical relative to its midship section [2].

Later, when he was working out practical methods of calculating the parameters of heaving and pitching on the basis of solving the problem for a cylindrical ship, in reference [3] M.D.Khaskind permitted inaccuracy to creep in when he assumed that the potential of the induced speeds, when heaving and pitching were made to occur in still water, do not depend on x. The result of this was that the type (2) term was missing from his equation for the hydrodynamic forces.

For a cylindrical ship we can write:

$$\varphi_1 = \varphi_1(y,z); \quad \varphi_2 = \varphi_1 x; \qquad dS = dx \cdot dl, \tag{3}$$

where dl is an element of the outline of frame station $\mathcal{L}$.

On the basis of equations (3), we get the following expression from equation (2) for the intensity of the additional running load:

$$\Delta q(x) = \frac{\Delta z}{dx} = -\rho\, \omega\, u e^{i\sigma t} \int_{\mathcal{L}} \varphi_1 \frac{\partial \varphi_1}{\partial n}\, dl. \tag{4}$$

For pitching we have:

$$\omega e^{i\sigma t} = \dot{\psi}. \tag{5}$$

Following the reasoning of M.D.Khaskind [3], we further write:

$$-\rho \int_{\mathcal{L}} \varphi_1 \frac{\partial \varphi_1}{\partial n}\, dl = \mu_{33}(x) - \frac{i}{\sigma} \lambda_{33}(x), \tag{6}$$

where $\mu_{33}(x)$ and $\lambda_{33}(x)$ are the running connected mass and the coefficient of damping.

On the basis of equations (5) and (6), we get the following from equation (4):

$$\Delta q(x) = u\dot{\psi}\left[\mu_{33}(x) - \frac{i}{\sigma} \lambda_{33}(x)\right]. \tag{7}$$

Noting also that

$$\psi = \psi_0 e^{i\sigma t}, \quad \dot{\psi} = i\sigma\psi,$$

instead of equation (7) we get:

$$\Delta q(x) = u\dot{\psi}\mu_{33}(x) + u\psi\lambda_{33}(x). \tag{8}$$

Let us clarify the physical sense of expression (8) for the intensity of the additional hydrodynamic load.

Let us consider the section at a distance x from the midship section, with a difference $\psi > 0$ (Fig. 3). We have assumed that the ship has a horizontal speed u = const., which results in the additional transverse section speed (in the plane of this section)

$$\Delta v = u\psi, \tag{9}$$

and the additional acceleration relative to the surface of the water

$$\Delta\dot{v} = u\psi. \tag{10}$$

If we compare equations (9) and (10) with equation (8), we find that equation (8), and also of course equation (2), allow for the effect of the additional speed and acceleration of the transverse section of the ship relative to the surface of the water, this speed and acceleration developing while the ship is moving forwards, on the hydrodynamic forces.

In order to estimate the effect exerted by the additional running load (8) on bending the hull of the ship when waves are encountered, let us consider the heaving and pitching of the symmetrical midship plane of a ship.

The total intensity of the load in the section x is determined by means of the equation

$$\left.\begin{aligned} q(x) = -[m(x) + \mu_{33}(x)](\ddot{\zeta} + x\ddot{\psi}) - \lambda_{33}(x)(\dot{\zeta} + x\dot{\psi}) + \mu_{33}(x)u\dot{\psi} - \\ -\gamma b(x)(\zeta + x\psi) + \lambda_{33}(x)u\psi + \gamma r_0 b(x)\varkappa_2(k_0T)[1 - \frac{\mu_{33}(x)\sigma_0^2}{\gamma b(x)}\cos(\sigma 1 + k_0 x), \end{aligned}\right\} \tag{11}$$

where, in addition to the symbols used before, we introduce the following:

b(x) is the width of the waterline at section x;

$\varkappa_2(k_0T)$ is a correction for the smoothing of wave movements as the depth increases;

$k_0 = 2\pi/\lambda$, the wave number.

If we write the heaving and pitching equation in the form

$$\int_{-0.5L}^{0.5L} q(x)dx = 0, \qquad \int_{-0.5L}^{0.5L} q(x)xdx = 0, \tag{12}$$

and take into account the symmetry of the ship, we get:

$$(M+M_1)\ddot{\zeta} + \lambda_\zeta\dot{\zeta} + \gamma F\zeta - M_1 v\dot{\psi} - \lambda_\zeta v\psi = 2r_0\varkappa_2(k_0T)\int_{-0.5L}^{0} b(x)\left[1 - \frac{\mu_{33}\sigma_0^2}{\gamma b(x)}\right]\cos k_0xdx \cos\sigma t;$$

$$(K + K_1)\ddot{\psi} + \lambda_\psi\dot{\psi} + \gamma J_y\psi = -2r_0\int_{-0.5L}^{0} xb(x)\left[1 - \frac{\mu_{33}\sigma_0^2}{\gamma b(x)}\right]\sin k_0xdx \sin\sigma t.$$

It follows fom equation (13) that the equations for the heaving and pitching of a symmetrical ship prove to be linked when the additional forces depending on the forward speed are taken into account. There is a relationship between heaving and pitching, and for a symmetrical ship there is no reciprocal connection.

If we write the second of equations (13) in the form

$$\ddot{\psi} + 2\gamma_2 n_2\dot{\psi} + n_2^2\psi = \frac{1}{2}\frac{\gamma r_0 BL^2}{K + K_1}\theta\, ie^{i\sigma t}, \tag{14}$$

where $n_2^2 = \dfrac{\gamma J_y}{K + K_1}$, $\qquad 2\gamma_2 = \dfrac{\lambda_\varkappa}{(K + K_1)n_2}$,

$$\theta_1 = \varkappa_2(k_0T)\int_{-1}^{0}\xi\frac{b(\xi)}{B}\left[1 - \frac{\mu_{33}\sigma_0^2}{\gamma b(\xi)}\right]\sin\pi\frac{L}{\lambda}\xi d\xi, \text{ and } \xi = \frac{2x}{L},$$

we find its solution

$$\psi = (\psi_1 + i\psi_2)e^{i\sigma t}. \tag{15}$$

We can quite easily write the following equations for ψ_1 and ψ_2:

$$\psi_1 = \frac{1}{2} r_0 \frac{BL^2}{\mathfrak{I}_y} \theta_1 \alpha_1,$$

$$\psi_2 = \frac{1}{2} r_0 \frac{BL^2}{\mathfrak{I}_y} \theta_1 \alpha_2; \tag{16}$$

here $\alpha_1 = \dfrac{2\gamma_2 d_2}{(1-d_2^2)^2+4\gamma_2^2 d_2^2}$, and $\alpha_2 = \dfrac{1-d_2^2}{(1-d_2^2)^2+4\gamma_2^2 d_2^2}$ are the

dynamic coefficients.

On the strength of equations (15) and (16), the first of the equations (13) can be transformed as follows:

$$\ddot{\zeta} + 2\gamma_1 n_1 \dot{\zeta} + n_1^2 \zeta = \frac{\gamma r_0 BL}{M + M_1} [\theta + v(R_1 + iR_2)] e^{i\sigma t}, \tag{17}$$

where

$$\left.\begin{aligned} n_1^2 &= \frac{\gamma F}{M + M_1}; \qquad 2\gamma_1 = \frac{\lambda \xi}{(M + M_1) n}, \\ R_1 &= \frac{1}{2}\theta \frac{n_1 L(M + M_1)}{\gamma \mathfrak{I}_y} (2\gamma_1 d_1 + \alpha_2 d_1) \frac{1}{1 + \frac{M}{M_1}}, \\ R_2 &= \frac{1}{2}\theta \frac{n_1 L(M + M_1)}{\gamma \mathfrak{I}_y} (2\gamma_1 d_2 + \alpha_1 d_1) \frac{1}{1 + \frac{M}{M_1}}, \\ \theta &= \varkappa_2(k_0 T) \int_{-1}^{0} \frac{b(\xi)}{B} \left[1 - \frac{\mu_{33}\sigma_0^2}{\gamma b(\xi)}\right] \cos \pi \frac{L}{\lambda} \xi \, d\xi; \quad d_1 = \frac{\sigma}{n_1}. \end{aligned}\right\} \tag{18}$$

The solution to equation (17) can be found in the form

$$\zeta = \zeta_0 e^{i\sigma t}, \tag{19}$$

while

$$\zeta_0 = \zeta_{01} + \zeta_{02},$$

where

$$\zeta_{01} = r_0 \frac{1}{\alpha} \theta (\beta_1 + i\beta_2) \tag{20}$$

is the complex amplitude of heaving and pitching without allowing for the additional forces associated with the ship's forward speed;

$$\zeta_{02} = r_0 \frac{1}{\alpha} u \left[R_1 \beta_2 + R_2 \beta_1 + i(R_2 \beta_2 + R_1 \beta_1)\right] \qquad (21)$$

is the additional complex amplitude of heaving, proportional to the speed u.

In equations (20) and (21) we have used the following symbols:

$$\beta_1 = \frac{2 \gamma_1 d_1}{(1 - d_1^2)^2 + 4 \gamma_1^2 d_1^2}, \qquad \beta_2 = \frac{1 - d_1^2}{(1 - d_1^2)^2 + 4 \gamma_1^2 d_1^2} \qquad (22)$$

and α is the waterline coefficient.

The effects of the additional forces considered above on the amplitude of heaving can be estimated from the magnitude of the amplitude ratio $|\zeta_{02}| : |\zeta_{01}|$.

On the basis of equations (16)-(22), we get the following equation for a symmetrical ship:

$$k = \frac{|\zeta_{02}|}{|\zeta_{01}|} = \frac{1}{2} \frac{\theta_1}{\theta} \frac{L(M + M_1) v n_1}{(1 + \frac{M}{M_1}) \gamma \vartheta_y} \frac{\sqrt{d_1^2 + 4 \gamma_1^2}}{\sqrt{(1 - d_2^2)^2 + 4 \gamma_2^2 d_2^2}}. \qquad (23)$$

We can see from equation (23) that as the pitching comes closer to resonance ($d_2 = 1$), the ratio k increases.

To obtain a quantitative estimate, the ratio k was calculated for a symmetrical model which had been tested for deflection in the tank at the Leningrad Shipbuilding Institute (LSI). The dimensions of the model were: L = 2.08 m; B = 0.32 m; T = 0.112 m; α = 0.67; D = 49.8 kg.

Two cases of loading the model were considered:

still water bending $M_{sw} = -\frac{1}{60} DL$;

still water flexing $M_{sw} = \frac{1}{60} DL$.

Table 1 contains the results of calculating the ratio (23) for four towing speeds.

TABLE 1

u, m/sec	0.4	0.8	1.2	1.6
$Fr = \frac{u}{\sqrt{gL}}$	0.089	0.178	0.266	0.356
Bending in still water, k	0.412	1.45	2.15	1.88
Flexing in still water, k	0.494	1.33	1.39	1.23

It follows from Table 1 that, if allowance is not made for the additional forces considered earlier, the error in the magnitude of heaving beomes extremely great.

If we convert to real quantities in equations (16) and (22), we obtain the solution, as regards the heaving and pitching of a symmetrical ship, in the following form:

$$\psi = -\frac{r_0}{L}\frac{BL^3}{2\mathfrak{J}_y}\theta_1\alpha_\psi \sin(\sigma t - \delta_\psi), \qquad (24)$$

$$\zeta = r_0 \frac{\theta}{\varkappa}\alpha_\zeta [\cos(\sigma t - \delta_\zeta) - k \cos(\sigma t + \delta - \delta_\zeta)], \qquad (25)$$

where

$$\alpha_\psi = \frac{1}{\sqrt{(1-d_2^2)^2 + 4\gamma_2^2 d_2^2}}, \quad \alpha_\zeta = \frac{1}{\sqrt{(1-d_1^2)^2 + 4\gamma_1^2 d_1^2}},$$

$$\tan\delta_\psi = \frac{2\gamma_2 d_2}{1-d_2^2}, \qquad \tan\delta_\zeta = \frac{2\gamma_1 d_1}{1-d_1^2}, \qquad (26)$$

$$\tan\delta = \frac{d_1 \sin\delta_\psi + 2\gamma_1 \cos\delta_\psi}{d_1 \cos\delta_\psi - 2\gamma_1 \sin\delta_\psi}.$$

In order to estimate the effects of the additional forces associated with the speed u on the wave bending moment, we calculate this for the midship section of a symmetrical ship. The midship section bending moment is equal to

$$M(0) = - \int_{-0.5\,L}^{0} q(x)\,x\,dx. \tag{27}$$

If we substitute equation (12) in equation (27), and use equation (13) for the pitching and heaving of a symmetrical ship, we get:

$$M(0) = \frac{M}{2}(l_1 - l_2)\cdot(\sigma^2 \zeta_1 \cos\sigma t - \sigma^2 \zeta_2 \sin\sigma t) + \frac{M_1}{2}(l_1' - l_2)[(\sigma^2 \zeta_1 - u\sigma\psi_2)\cos\sigma t - (\sigma^2 \zeta_2 - u\sigma\psi_1)\sin\sigma t] - \frac{\lambda_\zeta}{2}(l_2' - l_2)[(-\sigma\zeta_2 - u\psi_1)\cos\sigma t + (-\sigma\zeta_1 - u\psi_2)\sin\sigma t] + r_0 \cos\sigma t \int_{-0.5\,L}^{0} \chi_2(k_0 T)[\gamma b(x) - \mu_{33}\sigma_0^2](x - l_2)\cos k_0 x\,dx, \tag{28}$$

where in addition to the symbols already employed, we have also used

$$l_1 = \frac{2\int_{-0.5\,L}^{0} m(x)\,x\,dx}{M}, \qquad l_2 = \frac{2\int_{-0.5\,L}^{0} b(x)\,x\,dx}{F},$$
$$l_1' = \frac{2\int_{-0.5\,L}^{0} \mu_{33}(x)\,x\,dx}{M_1}, \qquad l_2' = \frac{2\int_{-0.5\,L}^{0} \lambda_{33}(x)\,x\,dx}{\lambda_\zeta}. \tag{29}$$

Using equations (16), (20) and (21), we find

$$M(0) = M_1(0) + M_2(0), \tag{30}$$

where $M_1(0)$ is the midship section bending moment, calculated not allowing for the effects of the additional forces associated with the ship's speed;

$M_2(0)$ is the bending moment proportional to the forward speed of the ship.

The moment $M_1(0)$ can be calculated using the equation given in reference [4].

The bending moment $M_2(0)$ is:

$$M_2(0) = \frac{1}{4}\gamma r_0 B L^2 \theta_1 \frac{F L^2}{\mathfrak{I}_y}\frac{u}{n_1 L}[(A_2 B_2 - A_1 B_1)\cos\sigma t + (A_2 B_1 + A_1 B_2)\sin\sigma t], \tag{31}$$

where

$$
\left.
\begin{aligned}
A_1 &= \frac{M_1}{M}\,\frac{|l_1'| - |l_2|}{L}\,\frac{1}{1 + \frac{m_1}{M}}, \\
A_2 &= \frac{l_2' - l_2}{L}\,\frac{2\gamma_1}{d_1}, \\
B_1 &= \frac{1}{1+\frac{M_1}{M}}\,\frac{d_1^2(1-d_1^2)(2\gamma_1\alpha_1 + \alpha_2 d_1) + 2\gamma_1 d_1^3(2\gamma_1\alpha_2 + \alpha_1 d_1)}{(1-d_1^2)^2 + 4\gamma_1^2 d_1^2} - \alpha_2 d_1, \\
B_2 &= \frac{1}{1+\frac{M_1}{M}}\,\frac{d_1^2(1-d_1^2)(2\gamma_1\alpha_2 + \alpha_1 d_1) + 2\gamma_1 d_1^3(2\alpha_1 d_1 - \ _2 d_1)}{(1-d_1^2)^2 + 4\gamma_1^2 + d_1^2} - \alpha_1 d_1.
\end{aligned}
\right\} \quad (3
$$

Equation (31) oan be reduoed to a form oonvenient for its use:

$$\overline{M_2(0)} = \left|\overline{M_2(0)}\right| \cos(\sigma t - \varepsilon_2), \qquad (33)$$

where $\overline{M_2(0)} = \dfrac{M_2(0)}{\frac{1}{4}\gamma r_0 B L^2}$, the dimensionless bending moment.

The moment $M_1(0)$ oan be written in a similar form.

Table 2 oontains values of the dimensionless amplitudes of the bending moments $|\overline{M_1(0)}|$ and $|\overline{M_2(0)}|$, also the phase angles ε_2 oaloulated for the model the dimensions of whioh were given earlier.

TABLE 2

	u, m/sec	0.4	0.8	1.2	1.6
	$Fr = \frac{u}{\sqrt{gL}}$	0.089	0.178	0.266	0.356
$M_{sw} < 0$	$\lvert\overline{M_1(0)}\rvert$	0.0448	0.0391	0.0402	-
	$\lvert\overline{M_2(0)}\rvert$	0.00272	0.00458	0.0020	0.0013
	$\lvert M_2(0)\rvert : \lvert M_1(0)\rvert$	0.06	0.12	0.05	-
	ε_2^0	-23	28	63	14
$M_{sw} > 0$	$\lvert\overline{M_1(0)}\rvert$	0.0435	0.0448	0.047	-
	$\lvert\overline{M_2(0)}\rvert$	0.0035	0.0033	0.0012	0.0010
	$\lvert M_2(0)\rvert : \lvert M_1(0)\rvert$	0.08	0.07	0.02	-
	ε_2	-3.0	79	79	19

It follows from Table 2 that, whatever the pitching and heaving conditions, the amplitude of the additional bending moment $M_2(0)$ is small by comparison with that of the moment $M_1(0)$. The amplitude of the moment $M_2(0)$ depends very largely on the magnitude of A_1, calculated using equation (32). In the case under consideration the parameter A_1 had the following values:

$$A_1 = -0.0337 \quad \text{with} \quad M_{sw} = -\frac{1}{60}DL;$$

$$A_1 = 0.0021 \quad \text{with} \quad M_{sw} = -\frac{1}{60}DL.$$

The particular example considered does not enable exhaustive conclusions to be drawn regarding the quantitative effect of the additional forces associated with the forward speed of a ship on the bending moment. One would, however, expect this effect to be appreciable in the case of a ship with a bend in still water.

If the flexing moment in still water $\left(\frac{M_{sw}}{DL} > \frac{1}{60}\right)$ is substantial, the effects of the additional forces mentioned above on the wave bending moment in the midship section will always be infinitesimal (of the order of 1.0-3.0% of $|M_1(0)|$).

On the whole, the solution considered above has shown that heaving, and under certain conditions the wave bending moment in the midship section, are greatly related to the forces proportional to the forward speed of the ship, and these forces cannot be ignored when the conversion functions for heaving, pitching and the bending moment are being determined.

It must, at the same time, be noted that the solution obtained above is only effective for a ship moving at a constant speed. In actual fact, when a ship is passing through waves, its speed alters periodically owing to the effects of pitching and heaving, the wave train, and the wind. The quantitative estimates made above may be appreciably altered owing to this. The plan for further developing the problems considered in this article includes the extremely important task of solving the problem of pitching and heaving for a ship moving with a constant restraint, this being a first approximation to the actual conditions under which ships sail.

In conclusion, let us write equations for the heaving and pitching of a ship asymmetrical relative to its midship section.

After substituting equation (11) in equations (12), we get:

$$\left.\begin{aligned}(M+M_1)\ddot{\zeta}+\lambda_{\zeta}\dot{\zeta}+\gamma F\zeta+S_{M+M_1}\ddot{\psi}+[\lambda_{\zeta\psi}-uM_1]\dot{\psi}+(\gamma F-\lambda_{\zeta}u)\psi=F_0\cos(\sigma t+\delta_{\zeta})\\(K+K_1)\ddot{\psi}+(\lambda_{\psi}-uS_{M+M_1})\dot{\psi}+(\gamma \mathfrak{J}_y-\lambda_{\zeta\psi}u)+S_{M+M_1}\ddot{\zeta}+\lambda_{\zeta\psi}+\gamma S_F\zeta=M_0\cos(\sigma t+\delta_{\psi})\end{aligned}\right\}\quad(34$$

The comparison of equations (34) and (1) shows that allowing for the additional forces associated with the ship's forward speed only greatly alters the extent to which heaving is related to pitching. The terms in the second of equations (34) defining the effects of heaving on pitching coincide precisely with the corresponding terms in equations (1). Consequently the relationship of pitching to heaving remains extremely indefinite. By comparison with equations (1), certain of the terms in the equation for pitching are slightly different in equations (34). From the quantitative point of view, however, these changes are extremely small (not more that 10% of the corresponding basic term). The principal features of the combined effects of the angular and vertical movements of ships during heaving and pitching, which can be found by analysing [illegible] equations (34), coincide precisely with the results of the experiments mentioned earlier (Fig. 2).

When the order of the different terms in equations (34) is assessed, it is found that the surging of an asymmetrical ship can be calculated, with an error not exceeding 10%, using equations (13), on condition that the right hand sides of these equations are replaced by the following expressions:

in the equation for heaving,

$$r_0\,\varkappa_2(k_0T)\int_{-0.5L}^{0.5L} b(x)\left[1-\frac{\mu_{33}(x)\,\sigma_0^2}{\gamma b(x)}\right]\cos(\sigma t+k_0x)\,dx\;;\qquad(35)$$

in the equation for pitching,

$$r_0 \varkappa_2(k_0T) \int_{-0.5 L}^{0.5 L} b(x)[1 - \frac{\mu_{33}(x)\sigma_0^2}{\gamma b(x)}] \cos(\sigma t + k_0 x)xdx. \qquad (36)$$

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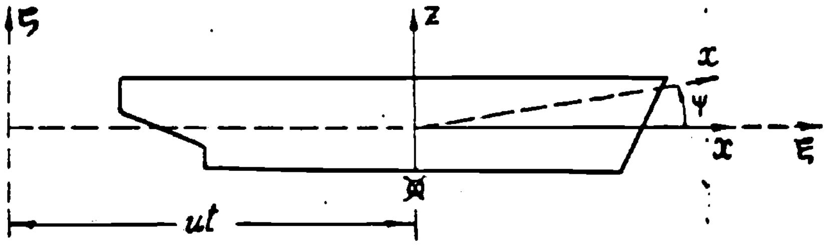
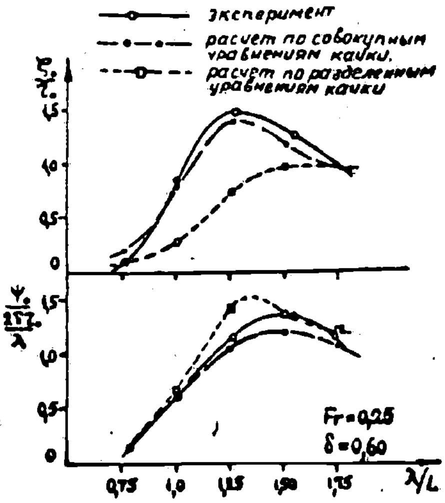


Fig. [redacted].I



Эксперимент = experiment

Расчет по совокупным уравнениям качки = calculation by combined equations for heaving and pitching.

Расчет по разделенным уравнениям качки = calculation by the equations for heaving and pitching separately.

Fig. 2.

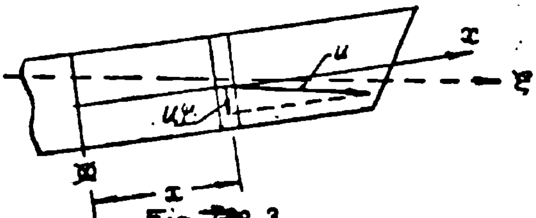


Fig [redacted].3