REPORT OF RESISTANCE COMMITTEE
APPENDIX 5

SHIP RESISTANCE IN SHALLOW AND RESTRICTED WATER

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Publications in the field of shallow and restricted water effects on ship resistance are comparatively scarce, and little systematic attention has been paid to these effects in the history of the ITTC. With the effects of incipient shallow and restricted water becoming of growing importance, in the form of trial trips on insufficiently deep measured miles and model tests with too big models in too small tanks, it seemed that it would be useful to review the state of the art in this field of ship hydrodynamics. The present report aims to give such a review and it deals with most of the literature that has appeared in the years between 1940 and 1972.

2. Shallow water effects

The first systematic approach to the problem of the effect of shallow water on resistance was made by Schlichting (52). From six series of tests with geosims at equal relative water depths Schlichting determined curves of resistance per ton displacement for constant Reynolds number and varying Froude number. In this way he could eliminate the effect of viscosity. The effect of tank width was eliminated by comparison of the curves for the different model sizes. The resistance curves so obtained for different water depths were analysed on the basis of the assumption that shallow water affects the non-viscous ship resistance in two ways:-

(a) the wave system of the ship is modified due to the difference in the relationship between wave length and wave speed in deep and in shallow water. This phenomenon is known as the wave retardation effect

(b) the average "effective" potential flow speed around the ship is modified due to the presence of the bottom, which is called the backflow effect.

If it is assumed that the wave resistance of a ship in deep or shallow water depends only on the length of the generated waves, the difference between the effective speeds in deep and shallow water at which equal wave resistance is experienced can be calculated easily. On base of this assumption it is possible, therefore, to convert a wave resistance curve for a ship or model in deep water into the wave resistance curve for the same ship or model in shallow water. The latter curve, however, shows wave resistance on base of the effective speed described above, which differs from the actual ship or model speed.

Schlichting determined the latter speed difference from his test series. Since it is in the first instance a result of a modification of the "primary" potential flow, he expected that the relative speed difference would not change very much with speed. Furthermore Schlichting assumed the relative speed difference to the dependent on the parameter $\sqrt{Am}/h$ only and independent of any other ship form parameters.

The usefulness of both assumptions was confirmed by Schlichting's analysis of his model experiments, and a diagram could be set up enabling the above-mentioned speed corrections to be applied very easily. With this diagram the conversion of deep water resistance curves into shallow water resistance curves is reduced in principle to a simple routine procedure.

Weinblum (60) in a paper on the theoretical calculation of wave resistance in shallow water concluded that the wave retardation principle was a reasonable approximation. More recently attention was drawn to the fact that the results of Schlichting's method depend to a high degree on the way in which the viscous resistance is calculated, both in deep and shallow water. It was also pointed out by Lap (64), that the wave resistance of a ship is not only dependent on the wave length, but also on the wave height. The latter varies approximately as the square of the effective speed and therefore with the backflow corrections. Lap suggests therefore that the wave resistance for the same ships in deep and shallow water should satisfy a relation:-

$$R/\tfrac{1}{2}\rho V_{eff}^{2}A_m = f(\lambda/L)$$

instead of Schlichting's relation:-

$$R/D = f(\lambda/L).$$

Particularly when the Schlichting hypothesis is applied in combination with restricted width Lap's suggestion may lead to considerably different results. Landweber (53) calculated the backflow effect due to shallow water theoretically. The difference beween his results and the lower values found by Schlichting were attributed by Landweber to side-wall effects during Schlichting's tests.

3. Incipient shallow water effects

With the ever increasing size of ships the results of many measured mile speed trials begin to experience the effect of shallow water on resistance. In order to try to calculate these effects Lackenby (49) evaluated the Schlichting method described in paragraph 2 for small values of $\sqrt{A_m}/h$ and F_{nh}. From his calculations this author concludes that the backflow effect is negligible for

$$\frac{\sqrt{A_m}}{h} \leqslant 0.18$$

and that the wave retardation effect is negligible for

$$F_{nh} \leqslant 0.45$$

If both conditions are satisfied no speed or resistance corrections are necessary. This requirement is considerably more rigorous than the one published as a diagram by Kempf (66) and the one used by Schuster (65) for purposes of a different order of accuracy (see also paragraph 4).

More recently Scott (50) derived the effective speed corrections for the Schlichting method from a statistical analysis of trial trip results in deep and shallow water. In cases where the wave retardation effect could be neglected and $\sqrt{A_m}/h > 0.32$ he found higher values for the backflow corrections than those according to Schlichting-Lackenby. From part of the material used by Scott, a correction that was much smaller than the one according to Schlichting-Lackenby was derived by Canham and Clements (67).

4. Restricted water effects

Until about 1940 there has been considerable doubt as to the applicability of the Froude method to carrying out model experiments for ships in canals of restricted depth and width. It was the work of Helm (1) that finally put an end to this doubt by showing that apparent discrepancies between different test results could be attributed to the effects of tank width and to errors introduced by the use of a false bottom with slots along the tank walls. Helm demonstrated beyond doubt that test results obtained with models of different sizes in canals of equal relative dimensions could be converted into each other by applying the Froude method.

Helm also suggested intoducing the hydraulic radius of the canal section into the calculation of the depth Froude number in order to correlate test results for different tank widths according to the Schlichting method. The corrections of this method, already doubted by Weinblum (60) were invalidated by more recent research of Schuster (65).

The effect of the slots along false bottoms was interpreted by Helm as a virtual increase in water depth. In view of recent work it is very probable that it is physically more correct to consider this very important effect on model resistance as a virtual increase in tank width. In this respect work by Betts and Binnie (40) must also be mentioned. These authors described model tests in a water channel with a slotted wall. By special precautions uniform flow conditions in both transverse and longitudinal direction of the channel flow could be obtained. It appeared that the waves created by a fixed ship model were absorbed almost completely by the walls, if the highest slot in the side walls were just submerged. Comparatively large blockage factors seemed to cause little wall interference, and the wave profiles along the models compared very well with those found in a towing tank with much smaller blockage. In a later paper by Binnie and Cloughly (39) tests were reported that were carried out in such a slotted wall channel with blockage factors up to 0.05 and also in a very large towing tank. The differences in wave profile and model resistance proved to be small and could in general be attributed to

spontaneous waves in the water channel at Froude numbers exceeding 0.45. Measures were described to annul these waves, as a result of which the observations in channel and towing tank became very nearly identical.

Maybe it deserves attention to consider the development of more sophisticated slot configurations along false bottoms in towing tanks in order to simulate conditions that come very near to those in shallow water of unrestricted width. The hydraulic phenomena around ships or models in water of restricted width were approximately described on the basis of a one-dimensional theory for the first time by Kreitner (68). Assuming uniformity of water depth and speed in every cross-section and neglecting secondary wave phenomena, the flow conditions along an infinitely long ship in a canal are determined by Bernoulli's law and the continuity equation. Solution of these equations leads to a subdivision of the total speed range of the ship or model into three parts:-

(1) A sub-critical speed range, between zero speed and a speed $V_1 < \sqrt{gh}$ where both equations can be satisfied. This results in a hydraulic flow pattern with an excess speed beside the ship combined with a decrease in local water depth.

(2) A super-critical speed range between $V_2 > \sqrt{gh}$ and infinity where also both equations can be satisfied. In this case the solution is a hydraulic flow pattern with a reduced speed beside the ship combined with an increase in water depth.

(3) A critical speed range between V_1 and V_2, where no real solution exists, which means that it is physically impossible that all the water present in an undisturbed canal section passes along the ship.

At the time maybe Kreitner's work did not receive the attention it deserved, and it was not until 1952 that Schuster (65) published the results of his systematic investigations of ship models in a water channel at different drafts, depths and widths. From these

tests Schuster concluded that the effects of shallow water and blockage were indeed of a fundamentally different nature. The shallow water effect results from a deformation of the orbital paths of the water particles due to the waves of the secondary wave system. For normal canal dimensions and speeds it is not directly affected by the presence of side walls. On the other hand, the blockage effect causes a deformation of the primary wave system and thus a change in local water depths and speeds. It therefore affects the secondary wave system only indirectly in most cases.

According to Schuster (65) the shallow water effect on ship wave resistance is of little practical importance for $F_{nh} < 0.7$. Above this value the effect increases rapidly to re-ch a very high peak value when the local depth Froude number approaches unity. This phenomenon was also explained theoretically by a. o. Inui (69) and Maruo (70).

At the lower limit of the critical speed range the local depth Froude number has become unity everywhere alongside the ship. The width of the critical speed range increases, however, with blockage and as a result the wave resistance in restricted water reaches its maximum at a speed that decreases with increasing blockage. Correspondingly, in the whole sub-critical speed range the local depth Froude number, and therefore the wave resistance, increases with blockage.

According to Schuster (65) the wave resistance of a ship in a canal can be related to the deep water resistance, provided that the "effective" speed of the "undisturbed" flow is calculated according to Kreitner. A nomogram in the original paper facilities the calculation procedure for this correction. Wahab (71) applied this method to a number of tests with models at different water depths. He came to the conclusion that in the speed range $0.6 < F_{nh} < 0.9$, which is the most important in practice, the results are rather inaccurate. As possible causes he refers to Inui's (69) paper from which it would appear that the Schlichting hypothesis is too inaccurate. Also the exact resistance integral for ships in a canal as calculated by Kirsch (55) does not compare satisfactorily with Schlichting's wave retardation hypothesis.

More recently a preliminary analysis by Lap (72) according to the method mentioned in paragraph 2 led to very satisfactory results, if in the hydraulic calculations a correction coefficient was applied to the blockage factor and the viscous resistance was determined according to a three-dimensional method.

Another effect of blockage on resistance occurs in the critical speed range, that becomes wider as blockage increases. It is caused by the fact that the water in the canal, which cannot pass along the ship, is dammed up in front of it and experiences an increase in both potential and kinetic energy. This energy increase can simply be calculated from momentum considerations on the base of Kreitner's assumptions. This was done by Constantine (36), (37), who also described in detail the hydraulic phenomena around a ship moving in a canal. The front of the elevated water ahead of the ships forms a bore, which moves forward at a speed larger than the ship speed. The height of the bore increases at increasing ship speed, but this increase in height is accompanied by a decreasing speed difference between bore front and ship speed.

The presence of the bore causes the ship to trim by the stern. At the upper limit of the critical speed range the ship lies on the bore. Its speed is then equal to the speed of the bore front and the trim by the stern has become very great. At still higher speeds the bore disappears, since Bernoulli's theorem and the equation of continuity can be satisfied again. Behind the ship there exists in this speed region a deficiency of water as a result of which a wave of expansion is present there. The downstream part of this wave travels downstream with a speed $\sqrt{gh}$ and the upstream part at a lower speed.

Hooft (32) also described the flow in the three speed ranges and calculated the resistance due to the formation of the bore in front of the ship. According to him the frictional resistance has to be taken into account when defining the flow phenomena behind the ship. This would also explain the differences in resistance in the critical speed range as calculated by Hooft (32) and Constantine (35), (37).

Graff (36) has pointed out that within the critical speed range a considerable resistance may occur also due to the fact that not satisfying the continuity equation may be interpreted as the presence of a source distribution. This source distribution, when moving ahead, causes a resistance. Wave height measurements in Graff's paper generally confirm Schuster's findings. In particular it appears that the surface elevation is the highest at the bore front. Behind the front, which is always present over the complete canal width, waves are superimposed on the surface elevation. In the lower part of the critical speed range these waves do not reach the bow of the ship. In the higher part of that speed range these waves may cause instabilities in the condition of the ship.

Resistance measurements by Graff reveal that the maximum wave resistance is reached at depth Froude numbers between 0.90 and 0.93. The higher values apply to the smaller blockages and it could be imagined that this contradiction with Schuster's conslusion is caused by the fact that Graff did not calculate his depth Froude numbers with the local values of speed and water depth. Secondary wave phenomena in the supercritical speed range were described by Schuster (65), Graff (36), and Schmidt-Stiebitz (44). After the bore has disappeared a secondary wave system is generated with an opening angle that decreases from 90° at the lower limit of the supercritical speed range to zero as speed tends to infinity. Graff (36) has shown that for large values of this angle the waves reflected from the side walls may interfere with the original wave system, so that in this speed region the side walls may have a direct effect on wave resistance.

In the supercritical speed range there exists no equivalent of the Schlichting hypothesis. At the lower limit of this speed range the local depth Froude number beside the ship is still unity. Accordingly comparable secondary wave phenomena occur at higher speeds as blockage increases.

For details of methods for calculating the wave resistance in restricted water the reader is referred to earlier reports of the ITTC resistance committee (1966), (1969).

Much of the work in this field is based on the restricted water resistance integral as formulated by Sretensky (58).

Kirsch (55), (57) calculated the wave resistance in shallow and restricted water for a number of mathematical ship forms. The results are given in a form that is suitable for the determination of the wave resistance in restricted water from tests in deep water.

Ueno and Nagamatsu (56) investigated the effect of restricted water on the source distribution to be used to represent a certain ship form. Their conclusions confirm the results of Kirsch (46) discussed in paragraph 6, namely that for practical purposes the source distribution in restricted water may be taken the same as in unrestricted water. In principle, however, the source density representing the same ship form increases with the presence of canal walls and bottom. Ueno and Nagamatsu found also that in practical applications the effect of restricted water on wave resistance may be neglected as soon as canal width w exceeds 3/2L and depth exceeds 3/4L.

Newman and Poole (59) derived expressions for the wave resistance of pressure distributions moving with constant forward speed along the free surface of a canal. They also concluded that except for the vicinity of $F_{nh} = 1$ the effect of tank walls is not important for $w/L > 1$ to 2. The influence of several form parameters on the wave resistance is illustrated by a number of diagrams.

Kinoshita (27) analysed a large number of resistance tests covering all three speed ranges. Velocity measurements indicated that Kreitner's assumption of uniform flow over a cross-section was satisfied reasonably well. In his analysis Kinoshita assumed the wave resistance to be negligible and he used the local water speeds in the calculation of the viscous resistance. The residuary resistance so obtained was split up into an eddy-making resistance, which Kinoshita assumed to be proportional to the square of the relative increase in effective speed, and a resistance due to the formation of a bore. He also introduced a correction factor to convert nominal blockage into effective blockage, a proportionality coefficient in the above-mentioned relation for the

for the eddy-making resistance, a correction factor in the calculation of the energy of the bore and a fourth correction factor to be used in the critical speed range only and associated with a resistance component of which no clear physical description is given.

From the experiment series all these empirical correction factors could be determined. Graphs for these factors are included in the paper. With the exception of the eddy-making resistance factor, which appears to vary with C_p also, the correction factors are dependent on blockage only with reasonable accuracy.

Much experimental information on the bottom and wall effects on resistance, mainly in the sub-critical speed range, was also given by Comstock and Hancock (26). These authors did not attempt to analyse their results at that time. An important conclusion was, however, that the best value of the ratio width/depth for a deep water towing tank appears to be about 2.6.

5. Incipient restricted water effects

For various reasons the relative size of models to be tested in deep water-basins increases continuously, so that also in this case the effect on resistance of tank width and depth may no longer be neglected. A good deal of attention has been paid to this problem in recent years.

In various publications it was pointed out by Schuster (12), (14), (18) that in principle application of the Schlichting hypothesis and the Kreitner method should form the base of any method to correct model results for small blockage and wave retardation effects. According to Schuster for depth Froude numbers smaller than 0.7 corrections according to the Schlichting hypothesis can be omitted and for depth Froude numbers less than 0.3 and blockage factors less than 0.02 the Kreitner effect can be simplified to a speed correction calculated with the continuity law only.

Schuster applied his method to experiments by van Lammeren, van Manen and Lap (73) with Victory models in canals with blockage factors up to

0.025. Apart from oscillations in the experiment values the agreement between theory and experiment appeared to be good.

Hughes (19) in principle followed Schuster's line of though. However, he split up the Kreitner speed correction into a term caused by simple continuity considerations and a second term originating from change in depth and therefore dependent on both blockage and depth Froude number. Originally Hughes replaced the continuity term by a speed correction calculated theoretically by Lock (75) and further developed by Squire and Young (76) and expressed by the formula

$$\delta V/V = 0.48 \nabla/A^{3/2}$$

where ∇ is the volume displacement of the model and A is the tank sectional area.

This theoretically more correct speed correction applies to the speed in the neighbourhood of the body and is valid for towing tanks with a width - depth ratio of 2. Preliminary analysis of a large number of experiments revealed, however, that the pure Kreitner concept of using average flow speeds led to more consistent results. This seems to be in agreement with Kinoshita's (27) speed measurements alongside his models, at least for the higher blockage values.

Hughes' final analysis showed that the speed corrections derived from comparative tests with 72 models in two tanks with different cross-sections amounted on the average to twice the values found theoretically with the Schlichting-Kreitner method. Again this result seems to be in agreement with Kinoshita's values for the ratio ψ between the effective and geometrical blockage factors. Positive deviations of this ratio from unity could be imagined to arise from effects of non-uniform velocity distribution beside the model, negative deviations from variations of blockage factor along the length of the model. For all Kinoshita's models ψ approached a constant value slightly higher than 0.9 for blockage factors greater than 0.02. This would confirm Hughes' findings that the application of average speeds in the calculations leads to better results. For smaller blockage

values, however, γ increases rapidly with decreasing blockage, also in agreement with Hughes' conclusion that his correction factor K increases with decreasing model size in the same tank.

Hughes (19) and also Scott (11) attributed the high value of K to a Reynolds dependent effect of the tank boundaries on the viscous wake. The latter author developed a formula for speed corrections in which, like Hughes, he applied the Schlichting hypothesis in an approximated form for the wave retardation effect and the Squire and Young method for the backflow correction. The latter term was corrected, however, in two ways. In the first place a correction factor was derived in order to account for the surface depression that was not included in Squire and Young's calculations for a closed channel. Scott estimated this term by applying Bernoulli's law and amongst others by assuming an improbably high value of 1.3 for the ratio between the speed of the water alongside a ship in deep water and the ship speed. The second correction term allowed for a speed increase due to the presence of the wake. Scott assumed this term to be proportional to the cube of the ship length.

With the formula so obtained for a speed correction due to blockage Scott analysed the same experiments as Hughes (19), after making some corrections for drift velocities in the tank, and using more sophisticated statistical methods. His final results, when converted to Hughes' K factors, show considerable differences with the analysis by Hughes. According to Scott these differences are caused by Hughes' failure to use a valid weighting technique, and a drift bias at high model speeds.

In later publications Scott (16), (17) replaces both corrections of the Squire and Young formula by a single coefficient of proportionality, so that his ultimate speed correction for back flow is given by the formula:-

$$\delta V/V = b \nabla/A^{3/2}$$

A diagram is given for the values of b to be applied as a function of

BL^2/∇. This diagram was composed on base of a statistical analysis of a large number of tests.

For a number of results Scott compares measured blockage effects with values calculated according to various methods. His conclusions that his own method is by far the best might be affected by the fact that most of the experiment material used in the comparison seems to be used also for the composition of Scott's diagram of b-values.

When discussing results of such analyses attention has to be drawn to one of the conclusions in a paper by Hughes (15), namely, that apparent tank boundary interference values obtained from analyses relative to assumed viscous slopes may vary considerably with the value of the slope that is used.

Taniguchi and Tamura (77) also apply a speed correction in order to correct model resistance tests for blockage effects. The form of their formula

$$\delta V/V = 1.1\ m\ (L/w)^{3/4} \qquad \text{where } m = \text{blockage factor}$$

is derived from theoretical calculations for a combination of a source and sink advancing along the axis of a cylindrical tube, assuming that the distance between the source and the sink is not large relative to the diameter of the tube. This blockage correction also appears to be in good agreement with a large number of experiment results.

6. Local speeds and water depths around models in shallow and restricted water

In all the above conversion methods local speeds and water depths play an important part. Several authors therefore made a more theoretical approach to this problem.

Kirsch (46) demonstrated, on the basis of four examples of simple flow fields, that for the calculation of flow velocities in shallow and restricted water the same singularity distributions may be used as in deep water. The difference in form of the bodies produced by these distributions in restricted and unrestricted water appears to be negligible.

Tuck (78) calculated the disturbance of a uniform shallow water flow due to the presence of an immersed slender body. He derived formulae valid to the first order in slenderness to calculate wave resistance of and vertical forces on such bodies. The latter results were used to predict sinkage and trim at sub- and supercritical speeds. These predictions appeared to be in reasonable agreement with experiment results published by Graff, Kracht and Weinblum (80).

Similar calculations for critical speeds were made by Feldman and Lea (38) who also compared their results with the above-mentioned experiments. It appears that in this speed range sinkage and trim are dependent on Froude number, beam-depth ratio and midship section area. Sinkage and trim decrease for hulls which are more slender, have smaller draft-beam ratios and have larger beam-depth ratios. Dagan (33) also published some results of theoretical calculations for the two-dimensional free surface flow generated by a singularity moving with near-critical speed.

In a later publication Tuck (79) extended his method to include also the effect of restricted width. He found that the percentage increase in sinkage and trim as compared to unrestricted water is a unique function of $w/L.\sqrt{1-F_{nh}^2}$. Although Tuck sees no physical or mathematical reason for this result, a remarkable analogy can be found in a paper by Zierep (81), who proves that wall effect in windtunnels is governed by the parameter $w/L.\sqrt{1-M^2}$, where M is the Mach number. Other conclusions from Tuck's paper are that the effect of side walls increases with decreasing water depth and may never be neglected at near-critical speeds, and that at moderate values of depth Froude number the effect of side walls on trim is negligible for any width greater than the ship length. It would be interesting to examine whether this result means that for this and larger widths the effects of shallow water and restricted width may be considered completely separately and whether therefore the trim is mainly affected by shallow water phenomena.

Sturtzel, Graff and Müller (47) have done some experimental work in this field. Local water depths and velocities were measured in the vicinity of a ship model in restricted water. The results were in agreement with the changes in wave length of the wave systems due to shallow water, from which the conclusion might be drawn that indeed local values of speed and water depth have to be used when Schlichting's hypothesis is applied.

Sinkage measurements for seagoing tankers were reported by Sjostrom (61). From tests with a model of a 90,000 ton tanker diagrams were derived, where the sinkage fore and aft is plotted as a percentage of the beam. This coefficient appears to be dependent only on the depth Froude number. This is in agreement with earlier published results by Saunders (82) and more recent tests by Guliev (62). The latter author tested 8 Series 60 models in shallow water and in channels of various cross-sections. The sinkage found from these tests, when plotted as a percentage of the beam on base of depth Froude number, was found to be independent of the geometrical characteristics of the models concerned.

The relation between the sinkage and the changes in water depth around a ship was investigated by Sturtzel and Schmidt-Stiebitz (63). From an analysis of sinkage measurements, photographs of wave profiles along models and direct measurements of local water depths these authors concluded that in shallow and restricted water the sinkage is greater than can be expected on base of hydrostatic calculations only. The additional dynamic effects appear to vary with water depth, ship length and construction water line area.

Recently Dand (90) examined the sinkage and trim (or "squat") of full form ships in unrestricted shallow water, and described a semi-empirical method which predicts model squat with reasonably accuracy. The propeller action appeared to be important in that a self-propelled model trimmed less by the head than a towed model.

7. Viscous resistance

Experimental verification of all the conversion and calculation methods described in paragraphs 2 - 5 is dependent on the accuracy with which

the viscous resistance can be calculated. For ship forms in deep water it is now generally agreed that form factors have to be applied to any basic two-dimensional correlation line, although there exist differences of opinion as to their type and magnitude. In particular there is uncertainty about the behaviour of the viscous pressure resistance, as a consequence of which the question has to be put, whether the correct correlation line is represented by the line of zero Froude numbers in a C_T-R_n diagram.

This problem, not even solved for deep water, becomes of extreme importance for the extrapolation of test results obtained in shallow and restricted water. Under these circumstances the differences between the line $F_n = 0$ and the basic correlation line may quite well become four or five times as large as for the same model in deep water. As a consequence form factors determined from single model tests at non-wavemaking speeds may become extremely large. This results in a much steeper extrapolation and a much lower ship resistance prediction than would be found with conventional two-dimensional methods.

It seems to be of great importance, therefore, to know whether and how far the form factors thus determined are affected by the dimensions of a channel, and more particularly whether the large shifts in the position of the zero Froude number line may really be attributed to form effect in frictional resistance. Some of the classical methods applied in investigations in this field are the use of completely immersed double models, determination of viscous resistance from wake traverse analysis, determination of pressure resistance from pressure measurements on model hulls and finally tests with geosims and single resistance tests in the non-wavemaking speed range. Due to various causes many of these methods were never applied to ship models in restricted water.

Van Lammeren and Van Manen (3) published results of geosim tests obtained in pressure-tight channels of geometrically similar cross-sections. From these tests they concluded that the three-dimensional Lap-Troost extrapolator produced much better correlation between the different model sizes than the two-dimensional Schoenherr extrapolator.

For tank widths of practical importance the form factor did not vary appreciably with the width. The increase in form factor with depth was considerable, however. When the form factors were plotted to a base of blockage, it became clear that for blockage factors less than 0.02 the effect of blockage on the form factor becomes very small for any water depth.

Lap (6) has shown that, due to the increase of the relative boundary layer displacement thickness with Reynolds number, the form effect on frictional resistance caused by very shallow water should be expressed by a form factor that decreases with increasing Reynolds number. In the same paper Lap gives some more information about form factors for different ship forms in shallow and restricted water.

A diagram set up by Graff (83) based on a number of model experiments gives the increase in frictional resistance due to shallow water as a function of the draught-depth ratio T/h. Maximum increases of the order of 28 per cent are reached for $T/h = 0.9$. In a later paper by Graff and Müller (31) test results were analysed obtained with a geosim family in non-similar canal sections. The increase in frictional resistance was calculated by means of a method published by Horn (84), according to which the average excess velocity along a ship in deep water can be derived from the sinkage by applying Bernoulli's theorem. With this average speed increase these authors calculated the increase in frictional resistance, assuming Horn's method to be also valid for shallow and restricted water. The resistance increase was then split up into two parts. One part, which is also present in unrestricted water, was calculated according to Scholz (85).

The remainder was attributed to the effect of shallow water and compared satisfactorily with Graff's earlier published diagram. Relative tank width differences were not taken into account, but this seems to be permitted in view of Lap's diagrams.

The increase in viscous resistance was also determined by Graff from the low speed resistance of the same geosims as well as from the slopes

of the isofroudes. Both methods led to percentage increases that were roughly three times as high as the values calculated from the excess speeds. This brings the authors to the conclusion that two thirds of the viscous resistance increase is an increase in viscous pressure resistance.

Luthra (7) analysed the same model family, as well as another one that was also tested in non-similar canal sections. He compared the form factors due to shallow water with those according to Hughes for deep water and came also to the conclusion that the total form effect on viscous resistance is much larger than the magnitude that follows from Horn's method.

A large number of test results with individual models was analysed by Sturtzel and Graff (4) with respect to both resistance measurements at non-wavemaking speeds and sinkage measurements analysed according to Horn's (84) method. The viscous resistance form factors were corrected for the effect of tank width by means of a theoretical method described in the paper, according to which the model may be replaced by a vertical elliptical cylinder having the same length and displacement as the model concerned. After this width correction the depth effect was eliminated by extrapolating test results for a series of water depths to infinity on the basis of a linear relativity between the depth form factor and the parameter BTC_B/h^2 which was established in the paper.

The above parameter shows a remarkable resemblance to the widely used parameter $\sqrt{BT}/h$ introduced by Schlichting (52) as a parameter for backflow effects.

In this way Graff obtained deep water form factors from his shallow water tests, which show a reasonable agreement with similar values determined on deep water, but which are high as compared to theoretically calculated values. A similar procedure was followed in a more recent paper by Graff and Binek (30) on the resistance of a fast passenger ship at different depths of water.

Work of a completely different nature was carried out by Sturtzel and

Schmidt-Stiebitz (5) and continued later on by Schmidt-Stiebitz (48). These authors tried to gain an insight into the velocity distribution of the flow around a model in restricted water by locally roughening the hull surface and comparing the increase in resistance so obtained with tests with flat plates that were roughened in a similar way. The results, however interesting for other purposes, unfortunately do not contribute to our knowledge of the calculation of viscous resistance.

Not much is known about the roughness allowances to be applied to ships in shallow or restricted water. Van Lammeren and Van Manen (2) found negative trial trip allowances, however, for tests in very shallow water that were extrapolated according to Schoenherr. This again points in the direction of a too flat extrapolation curve. According to the experience of the Netherlands Ship Model Basin this phenomenon becomes excessive for large blockages combined with shallow water. This combination occurs frequently for tests of large barge fleets. Two-dimensional extrapolation of such tests leads to unacceptably high resistance predictions, resulting in large negative trial trip allowances and wrong propeller designs, because the propeller loading is overestimated. For this reason the N S M B has applied three-dimensional extrapolation according to the log A-method with the ITTC-1957 correlation line as a base for all shallow and restricted water work since the inauguration of its shallow water basin in 1958.

8. Interaction between resistance components

The very few available comparative test results of geosims in geometrically similar canals seem to indicate that the Froude principle may be applied for cases of not too extreme limitations in water depth and width. However, in the case of long, wide and comparatively slow models, such as large barge fleets, it may quite well occur that the gap between ship bottom and canal bottom is completely filled up by boundary layer flow. This phenomenon might explain the fact that for such models any conversion method based on the Schlichting-Kreitner hypothsis has failed completely so far. The question arises therefore as to how far in such cases the resistance equation may really be split up, as is usual, according to:-

$$C_T(R_n, F_n) = C_V(R_n, F_n) + C_W(R_n, F_n) = C_V(R_n) + C_W(F_n)$$

A serious need exists for geosim experiments with this type of ship, not only to determine how the presence of the bottom affects frictional resistance, but also to find out if viscous blockage between ship and canal bottom may influence the non-viscous part of the resistance.

9. Resistance in shallow flowing water

Normally in flowing water of small depth the flow velocity increases from zero at the bottom to a maximum at 80 - 90 per cent of the depth above the bottom. At the water surface the speed is then a little lower than the maximum. In model test work it is customary to define the model speed relative to the water as the difference between the model speed relative to the land and the maximum speed of the flowing water, taking into account the sign of the latter speed.

When a model moves downstream in such a velocity distribution the speed of the "undisturbed" flow relative to the model is greater than or equal to the relative water speed in still water. The opposite is the case when the model moves upstream.

Sturtzel and Schmidt-Stiebitz (21) have demonstrated experimentally the existence of these velocity differences in various locations in the flow field beside and under a ship model. The same experiments, as well as earlier tests by Helm (1) and later experiments by Graff (8), have shown that at equal speeds (as defined above) the resistance increases considerably during downstream navigation and decreases in a similar degree during upstream navigation. This phenomenon needs no further explanation in view of the relative velocities defined above.

The total resistance difference can be split up into a viscous resistance component and a wave resistance component. The magnitude of the latter was calculated theoretically by Kolberg (86), (24). The results of his numerical calculations were confirmed by model tests with a mathematical ship form published by Graff and Kolberg (22) and also by tests with other models. Kolberg's calculations led to a simple formula for the difference in speed for the same ship in flowing and still water at equal wave resistance. Kolberg assumed that at these corresponding speeds the generated wave systems would be similar. This assumption was confirmed by experiment for waves not too close to the model (2.5 to 3 model lengths).

Völker (23) tried to explain the difference in wave resistance qualitatively by the fact that the speed of waves of translation in flowing water varies with the height above the bottom. As a result of this, according to this author, the critical speed increases in upstream and decreases in downstream navigation. This conclusion is in full agreement with Kolberg's complete analytic treatment of the problem.

It seems therefore that the effect of flow distribution on wave resistance can be estimated theoretically with sufficient accuracy. Not much is known quantitatively however, about the viscous effects that also may be considerable in the speed range that is of practical importance. As was pointed out by Helm (87) and later on by Lap (88), there may exist, apart from the effect of the difference in relative potential flow velocity, an effect of the current on the velocity gradient in the boundary layer along the ship's bottom. This velocity gradient, according to these authors, is larger in downstream navigation than it is in upstream navigation.

In addition to all the above-mentioned effects a ship that navigates in flowing water always experiences a component of its weight in the direction of its motion which is no longer horizontal. This force is known as the fall resistance. It is positive in upstream navigation, and negative when navigating downstream; it can easily be calculated if the fall of the river is known.

10. Resistance of ships in a transverse current

Not much information is available on the resistance experienced by ships that move in transverse direction in shallow water. Referring to Callet (89) a very high resistance coefficient of $C_D = 6$ is often applied to tankers anchored in shallow water in a transverse current.

In order to verify this value Tryde and Joshi (41) did some experiments

with a flat plate perpendicular to the flow direction and with a draft equal to the water depth. The resistance coefficient of such a plate appeared to be dependent on Froude number. For Froude numbers less than 0.1 it approaches a constant value of 3, which is only half the earlier mentioned magnitude. For increasing values of the bottom clearance the resistance coefficient decreases.

An interesting method to obtain information about similar flow phenomena was published by Schmidt-Stiebitz (42). Half double models were tested, lying upside down on the bottom of a shallow water basin with flowing water. The occurring surface disturbances appeared to be in reasonable agreement with similar information obtained with models that move afloat in shallow water.

11. Shallow water model test procedures

The often very full forebody lines of shallow water vessels may cause a strong adverse pressure gradient in the flow along this part of the model which tends to reestablish laminar flow conditions at short distances behind the bow. For this reason it has become common practice in most shallow water laboratories to apply more effective turbulence stimulators than would be necessary for the same models, when tested in deep water basins.

A normal shallow water stimulator arrangement consists of two tripwires of 1.0 - 1.5 mm thickness, one at about 5 per cent of the model length behind the fore perpendicular, the other, depending on the ship form, at about 10 per cent or near the forward part of the parallel middle body.

Special attention has to be paid also to a very gradual acceleration of the models during the beginning of a test run. Depending on model size and acceleration waves of translation may be generated, that travel ahead of the model. When reflected to the tank ends these waves may seriously disturb any measurements taken during a later stage of the test run. Graff (36) has published some recordings of such waves. Much attention to similar phenomena was also paid by Schmidt-Stiebitz (20).

The Netherlands Ship Model Basin has installed special wave dampers in its shallow water basin, where frequently very big models of barge fleets have to be tested. In this way the time interval between test runs could be reduced from several hours to about 20 minutes.

In connection with the phenomena described above much tank length is needed in many cases in order to reach stable flow conditions around the models, even at sub-critical speeds. (see also paragraph 4).

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APPENDIX C

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