

Delft University of Technology  
SHIP STRUCTURES LABORATORY

**Ductile behaviour  
of cyclically in-plane compressed  
imperfect steel plate panels**

**Part II**

**TOLERANCES  
AND  
STATISTICAL MODEL  
FOR  
INITIAL PLATING DEFLECTIONS  
IN  
WELDED STEEL STRUCTURES**

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## SUMMARY

This report is on initial plating deflections of welded steel structures.

A new statistical model of the maximum amplitude of initial plate deflections, based on a careful regression analysis of 411 measurements, is proposed and compared with existing models.

The model predicts considerably higher cumulative values and calls therefore the validity of the limit tolerances for allowable maximum plate deflections into question. New tolerances are proposed.

# NOMENCLATURE

Roman symbols:	unit of measure
a - plate length	m
$a_{mn}$ - coefficient of the DTFS (the sine-sine part)	m
A - dimensionless constant	--
b - plate breadth	m
$b_{mn}$ - coefficient of the DTFS (the sine-cosine part)	m
B - dimensionless constant	--
c - constant	--
C - constant	--
$c_{mn}$ - coefficient of the DTFS (the cosine-sine part)	m
d - amplitude of a localized deflections (dent)	m
$d_{mn}$ - coefficient of the DTFS (the cosine-cosine part)	m
e - base of natural logarithm	--
E - Young's modulus	Pa
f - plate deflection measured according to Standards	m
I - number of the IPD measurement along plate length	--
J - number of the IPD measurement along plate breadth	--
k - parameter	--
l - distance between two points on the plate	m
L - gauge length	m
M - number of the DTFS coefficients along plate length	--
N - number of the DTFS coefficients along plate breadth	--
p - probability	--
q - equivalent plate deflection, used in Standards	m
s - plate skewing	m
t - plate thickness	m
u - the standard normal variable	--
w - plate deflections (surface)	m
W - matrix of equally spaced measurements of plate deflections	m
x - coordinate along plate length	m
y - coordinate along plate breadth	m
z - coordinate perpendicular to xy plane	m

## Greek symbols:

$\alpha$ - the significance level	--
$\beta$ - plate slenderness parameter	--
$\lambda$ - dent length (localized deflection length)	m
$\lambda_{mn}$ - constant of the DTFS	--
$\sigma_y$ - the yield stress	Pa
$\sigma$ - the standard deviation	Pa
$w$ - maximum amplitude of initial plate deflections	m

### Subscripts and superscripts:

O - initial  
b - bending  
d - dent  
e - refers to set of data  
F - refers to DTFS  
h - harmful  
i - index of the IPD measurement along plate length  
j - index of the IPD measurement along plate breadth  
m - index of the DTFS along plate length  
n - index of the DTFS along plate breadth  
max - maximum  
p - permissible  
r - refers to regression line  
s - skewing  
y - refers to the upper yield at strain rate of  $10^{-4}$  ("static")  
w - web

### Others:

$x|_{\text{condition}}$  - value of x for a given condition  
 $|x|$  - absolute value of x  
 $\bar{x}$  - the mean value of x

### Brackets:

braces  $g(x,y)$  - functional relation of x and y  
square brackets [x] - unit of measure of x  
[n] - number bibliographic reference

### Abbreviations:

DTFS - the Double Trigonometric Fourier Series  
IPD - Initial Plate Deflections -  $w_0(x,y)$   
MIPD - Maximum amplitude of IPD -  $w = \max(w_0(x,y))$   
DAMIPD - Dimensionless Absolute MIPD -  $|w|/t$   
LDAMIPD - natural Logarithm of DAMIPD -  $\ln(|w|/t)$   
SIPD - Shape of IPD  
SSL - Ship Structures Laboratory

# CONTENTS

Page

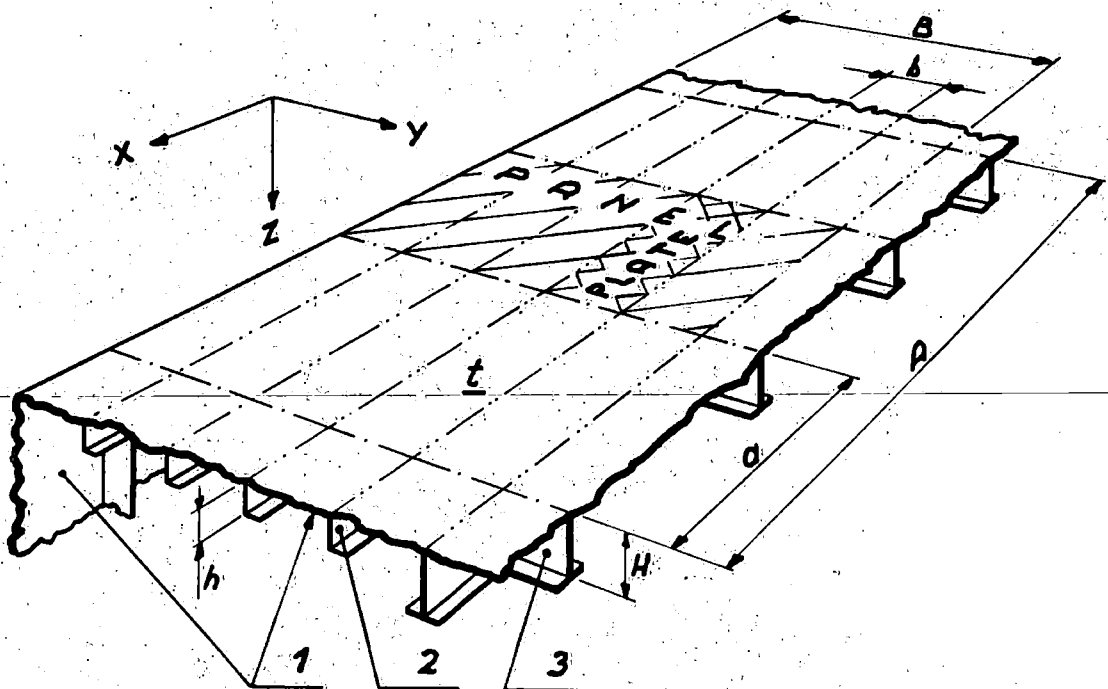
<b>SUMMARY</b> .....	iv
<b>NOMENCLATURE</b> .....	v
<b>DEFINITIONS</b> .....	ix
<b>1. INTRODUCTION</b> .....	1
1.1. GENERAL .....	1
1.2. EXAMPLES OF SPECIFICATIONS OF PERMISSIBLE PLATE DEFLECTIONS ACCORDING TO EXISTING STANDARDS .....	4
1.3. CRITICAL EXAMINATION OF THE PRESENT TREATMENT OF INITIAL PLATE DEFLECTIONS .....	8
1.4. THE AIMS .....	9
1.5. MEASUREMENTS OF INITIAL PLATE DEFLECTIONS ...	10
1.6. MATHEMATICAL DESCRIPTION OF INITIAL PLATE DEFLECTIONS .....	11
<b>2. MODEL OF MAXIMUM AMPLITUDE OF     INITIAL PLATE DEFLECTIONS</b> .....	13
2.1. GENERAL .....	13
2.2. REVIEW OF EXISTING MODELS .....	13
2.3. DISCUSSION OF THE MODELS .....	17
2.4. CHOICE OF IMPORTANT VARIABLES .....	18
2.5. POSTULATION OF THE MODEL .....	20
2.6. FITTING OF THE MODEL .....	24
2.7. EVALUATION OF THE MODEL .....	26
2.8. COMPARISON WITH OTHER MODELS .....	29
<b>3. PROPOSAL OF SPECIFICATION OF     PERMISSIBLE PLATE DEFLECTIONS</b> .....	31
3.1. GENERAL .....	31
3.2. PROPOSAL .....	32
<b>4. MODEL OF THE SHAPE OF INITIAL PLATE     DEFLECTIONS</b> .....	33
<b>5. CONCLUSIONS</b> .....	36
<b>6. ACKNOWLEDGEMENTS</b> .....	37
<b>BIBLIOGRAPHY</b> .....	38

## DEFINITIONS

Figure 1 shows an example of welded steel ship grillage and defines the terms for its components which are used in the present work. Figure 2 shows an example of the initial plate deflections and defines the A- and B-type amplitudes.

The other terms are defined as follows:

- initial** - refers to an un-straightened structural element, which is already a part of a completed section, block or whole structure, and before putting the structure into service (before the launching, in the case of marine structures).
- yield stress** - the upper yield stress determined in standard uniaxial tensile or compressive tests at a strain rate close to 0.0001 [strain/s]



- 1 - plating
- 2 - longitudinal stiffener
- 3 - transverse stiffener

Fig. 1. Example of a welded steel ship grillage and terms used.



# 1. INTRODUCTION

## 1.1. GENERAL

The initial plating deflections of welded steel ship grillages affect the design, production and performance of a ship. Among others, the following factors may be affected:

- use of thin plating during ship design,
- use of the ship,
- the aesthetic factors,
- the economic and technological process,
- strength.

The knowledge of a statistical model of the initial plating deflections may allow these effects to be foreseen and may be helpful in establishing or verifying their tolerances.

In the present work, initial plating deflections will be discussed taking only the uniaxial compressive strength into account, thus only in connection with the strength factors. Further, the subject matter will be restricted to initial deflections of single plate elements, i.e. to the Initial Plate Deflections (IPD) as shown in Figure 2. Some technological aspects of IPD in welded steel structures are given in [1].

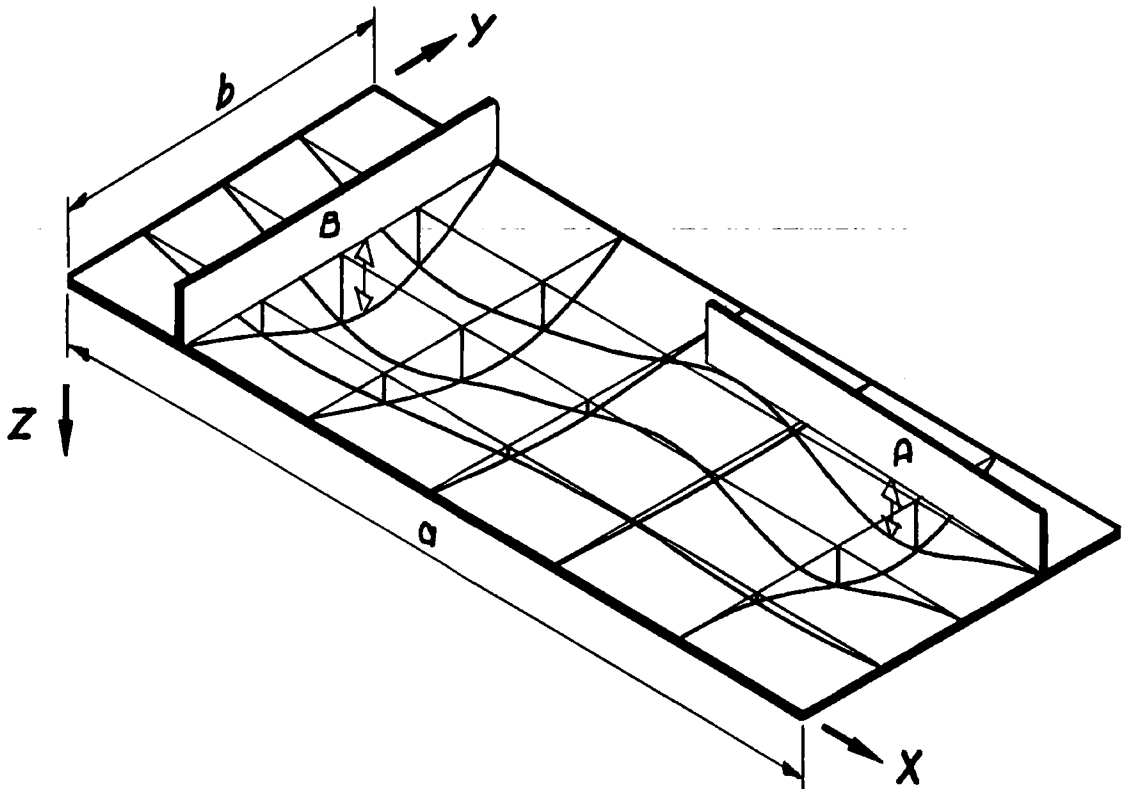


Fig. 2. Example of IPD with A- and B-type amplitudes indicated.

It is generally recognized that initial deformations affect the compressive strength [2]. However, the general statement: the larger the initial deflection amplitude the lower the strength valid in respect to columns is not valid in respect to plates (or shells). The reason is that the compressive strength of plates depends more on the geometry than on the maximum amplitude of the IPD. Section 4 gives more details.

A review of IPD and their tolerances is given in [3,4,5 and 6]. IPD are a result of many processes and events interfering with each other. Welding is the dominant source of IPD of ship grillages. Among the other sources, the buckling due to the forcing of the slender structure in order to eliminate gaps between its improperly cut or fitted elements should be mentioned here. Further, the IPD may include an isolated dent as a result of concentrated loads experienced during fabrication.

The shape and values of welding induced IPD depend on many factors; i.e. the welding parameters, welding sequence, geometric and material parameters of joined elements. As a result of this, IPD vary widely and they may be treated as a random variable.

This random nature of IPD allows the development of a mathematical model on the basis of the statistical analysis of appropriate data collections of IPD measurements. Of course, there are other possibilities, for instance a numerical simulation of the welding process, but they are outside the scope of the present work [7].

Several works presenting the results of measurements and statistical analysis of IPD have been published during the last fifteen years:

<u>year</u>	<u>investigator(s)</u>	<u>country</u>	<u>reference</u>	<u>measurements of:</u>
1975	Faulkner	England	[8]	max amplitude,
1975	Kmiecik and Czujko	Poland	[9 and 10]	3d-shape,
1977	Somerville	England	[11]	max amplitude <sup>1</sup> ,
1977	Reupke	W.Germany	[12]	max amplitude,
1977	Kringel	W.Germany	[13]	max amplitude,
1978	Czujko and Carlsen	Poland	[14]	3d-shape,
1979	Ivanov	Bulgaria	[15]	max amplitude,
1980	Fujita	Japan	[16]	max amplitude,
1980	Antoniou	Greece	[17]	max amplitude,
1984	Antoniou	Greece	[18]	3d-shape,
1985	Jastrzebski et al	Poland	[19]	3d-shape.

As can be seen the majority of investigators had only measured and analyzed the maximum B-type amplitudes of IPD in the geometrical plate centre or possibly in arbitrarily chosen points of the plates. The approach by itself supplies no information as to the shape of IPD.

<sup>1</sup> the plate deflections were measured along the longitudinal and the transverse plate axis at a distances of 1/4, 1/2 and 3/4 of "a" and "b" respectively.

An exception is the work carried out by Kmiecik and his team: Czujko till 1978, Jazukiewicz, Kulik, the author till 1983, and Jastrzebski; from the Ship Research Institute of the Technological University of Szczecin. Since 1973 they have been measuring and investigating the real three-dimensional shapes of IPD on thousands of ships' plates [9,10,14 and 19].

These measurements have been made using a long frame, equipped with movable electro-mechanical displacement gauge, which is placed longitudinally at a few positions over the plate breadth. More details are given in Section 1.5.

The author has contributed substantially to these works [19 and 20]: as a student measuring hundreds of IPD; proposing a new, improved description and measuring procedure for IPD, which includes initial deflections of stiffeners and plate skewing (See Section 1.6); and developing the data base of IPD.

This data base contains among other information: measurements of IPD, hull section data, geometrical data, plate and stiffeners material data, welding data, the straightening flag, the Fourier coefficients (equation 1.6.2), the plate skewing and maximum plate deflection.

The data base described here is unique. It contains variables which are important from the point of view of sources and the description of IPD. Depending on the purpose, the required data may be easily selected, processed and analyzed.

The new model for the maximum amplitude of IPD, presented later in this work, is based on statistical analyses of the data selected from the data base.

Antoniou et al [18], spurred on by the work of Kmiecik et al, published the second part of their research in 1984 which also included the analysis of the shape of IPD. They used a different procedure to measure plate deflection. Namely, they used a short frame equipped with three mechanical displacement gauges which was placed parallel to the short side of a plate, from 7 to 17 times, depending on the plate length. In such procedure, the plate deflection is measured in relation to possibly deflected long stiffeners. It is possible that the different methods might give different results, especially when applied to long plates, and that they are equivalent providing that the stiffeners supporting a plate are relatively straight in comparison with the deflections of the plate.

Before specifying the aim of the present work we will discuss the specifications of permissible plate deflections according to some existing standards.

## 1.2. EXAMPLES OF SPECIFICATIONS OF PERMISSIBLE PLATE DEFLECTIONS ACCORDING TO EXISTING STANDARDS

The existing specifications refer to an amplitude of IPD. First they define the way of measurement of the amplitude, then they give the permissible values, and finally they give guidance for corrections when the permissible value is exceeded.

In all specifications, the amplitude of IPD is measured using a gauge, and is identified as the maximum distance between the gauge and the plate surface. The specifications differ in required gauge length L (e.g.: L = b, 2b or l[m] ) and gauge position in reference to plate sides: parallel to long (A-type) or short (B-type) plate sides (see Figure 2).

The permissible values are given as a function of the hull region, the spacing (plate breadth), plate thickness and type of steel, i.e.: mild or higher tensile steel. Different combinations of these three parameters are represented. Most of the specifications give the permissible values as a function of the hull region only. Some specifications make a distinction between the standard permissible value and the limit permissible value. The standard value is intended to apply to 95% of plating in a particular area, i.e. not more than 5% of the plates are permitted to have deformations greater than the standard. The limit value is intended to apply to each single plate.

All specifications recommend fairing when the measured amplitude exceeds the permissible amplitude. Some specifications also specify temperature requirements when flame straightening is employed for fairing purposes, but this aspect falls outside the scope of this present work.

Table 1 [4,5,6,21,23,24 and 22] gives a summary of some existing IPD specifications and shows values of permissible IPD amplitudes for the following plate example: strength deck, mild steel, t = 16 [mm], b = 800 [mm],  $\sigma_y = 235$  [MPa]. ( 9.5 [mm]  $\approx$  3/8 [inch], 6.5 [mm]  $\approx$  1/4 [inch]).

Three representative specifications are given in detail below.

### Example I

Ship construction tolerances and defect correction procedures recommended by Lloyd's Register [23] are given in Hull Structures Report No. 84/38. This document has primarily to do with a Quality Scheme but, when a scheme is not in operation at a particular shipyard, the standards may be used by surveyors for guidance and in complementing their experience and judgement. The IPD tolerances recommended for new construction are as follows:

$$q = \frac{b}{c \sqrt{k}}$$

where: q - maximum plate deformation between adjacent stiffeners,  
b - plate breadth,  
c - parameter defined in Table 1.  
k = 1 for mild steels, k = 0.72 for higher tensile steels.

**TABLE 1**

Values of parameter  $c$  in the LR formula for permissible amplitudes of IPD

c		Item
standard	limit	
200	133	strength deck, shell plating, webs of primary members, all within 0.6L amidships,
120	80	all other plating

**Example II**

Bath Iron Works Inspection Guidelines specify the permissible maximum amplitude of IPD in the form of the following figure:

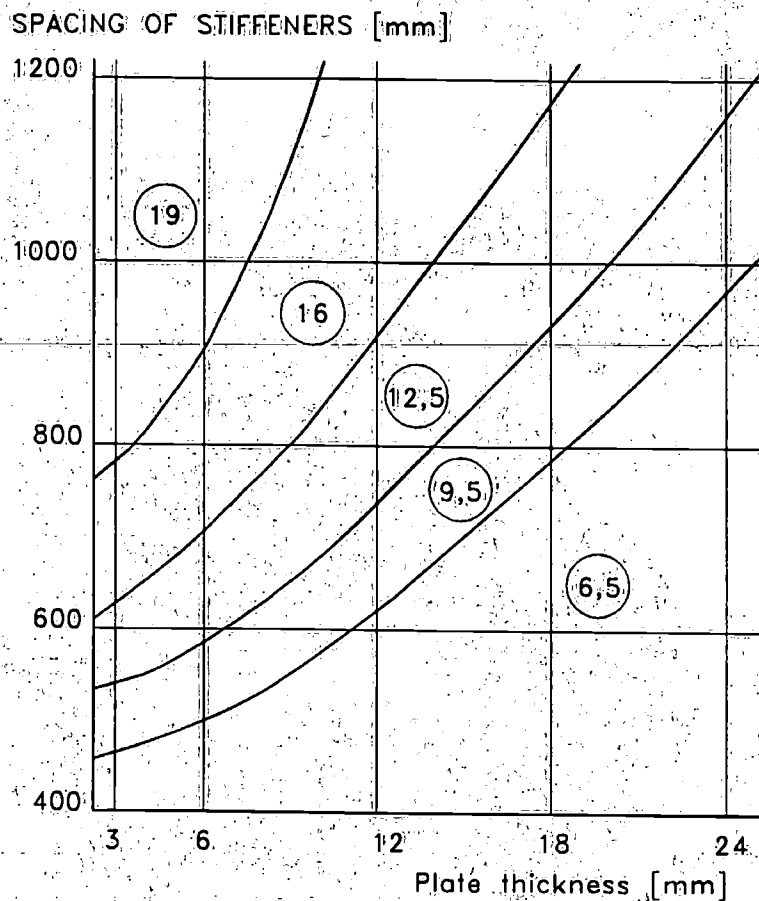


Fig. 3. BIW's permissible plate deflections in steel welded structures.

### Example III

In September 1988, the Dutch Normalization Institute issued the standard " Permissible deviations when aligning structural elements, welding and arranging the hull construction " [24], which is based on the practice used in Dutch shipyards, and two other standards:

- VIS 530 Accuracy in hull construction,  
(Sveriges Standardiseringskommission, Dec 1977)
- NS 6038 Marine industry- Accuracy in hull construction,  
(Norsk Verstedindustris Standardiseringsentral, 1978)

Table 2, after Section 6 of Standard, specifies a permissible deviation  $q_p$  of IPD, which is a function of hull region.

The deviation  $q$  of IPD should be calculated from the following relation:

$$q = \frac{L}{l} f$$

where:

$$L = 1000 \text{ [mm]}$$

$$L \geq l \text{ [mm]} \geq 300$$

$f$  - the greatest distance between gauge and plate,

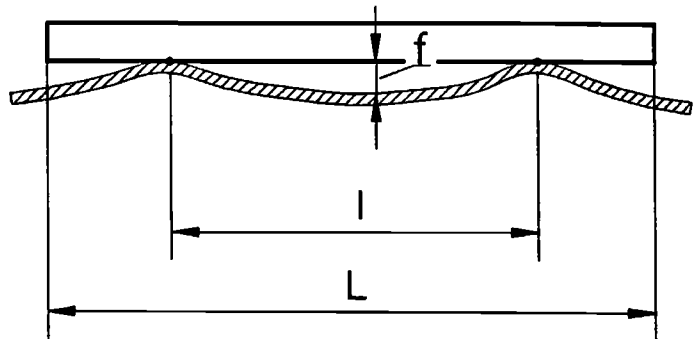


Fig. 4.

The standard does not clearly specify the position of the gauge in relation to the plate sides, when looking for maximum distance: parallel to short or to long sides. The author has assumed the last mentioned case (A-type).

TABLE 2

MEN's permissible amplitudes of IPD

$q_p$ [mm]	Item
8	strength deck, superstructure decks without deck-covering, shell plating within 40% of ship's length amidships, cargo hatch covers
10	visible deckhouse
15	all other plating

TABLE 3

Existing specifications of permissible amplitudes of IPD

Institution		Gage		Permissible deflection							
		length	position		function of				example		
Name	year		A-type	B-type	Item	b	t	b/t	$\sigma_y$	limit	standard
Gdańsk Shipyard (Poland)		b	.	x	x	.	.	.	.	7	.
Bath Iron Works (USA)		b	.	x	.	x	x	.	.	9.5	.
Sun Shipbuilding and Drydock Company		b	.	x	x	.	.	.	.	6.5	.
Swedish Shipbuilding Standards Centre	1976	1[m]	x	.	x	.	.	.	.	8	.
Det Norske Veritas	1977	b	.	x	.	x	.	.	.	8	.
British Standards ( a/b > 2 )	1980	2b	x	.	.	x	.	.	x	8	.
Lloyd's Register	1984	b	.	x	x	x	.	.	x	6	4
Japanese Shipbuilding Quality Standards	1985	b	.	x	x	.	.	.	.	6	4
German Shipbuilding Industry Standards	1985	b	.	x	x	.	.	.	.	6	4
Dutch Normalization Institute	1988	1[m]	x	.	x	.	.	.	.	8	.
Author	1990	b	.	x	.	x	.	x	.	14	6
	in preparation	b	x	.	.	x	.	x	.	-	-

### 1.3. CRITICAL EXAMINATION OF THE PRESENT TREATMENT OF INITIAL PLATE DEFLECTIONS

The setting of specifications is an evolving process. The overriding consideration during that process is to make the specifications as simple to apply as possible, because a surveyor does not have time to take extensive measurements. The question is whether existing specifications are not oversimplified and whether other more adequate simply formulation are possible.

As shown in the Section 1.2 the Classification Societies, shipyards and companies, in order to eliminate unfavourable effects, use tolerances of permissible distortions based on the maximum amplitude of IPD, which is measured using a gauge placed only along the length (type A) or only along the breadth (type B) of a plate.

On the one hand, the recent B-type tolerances permit higher maximum amplitudes of IPD than the A-type tolerances (see Table 3). Thus the use of the A-type tolerances for other purposes than for plates which compressive strength has to be taken into account, can lead for to unnecessary straightening.

On the other hand, the use of the B-type tolerances for plates for which the compressive strength has to be taken into account, was criticized by Czujko and Kmiecik [10]. They investigated the geometry of rectangular plates before (the requirement unsatisfied) and after fairing (the requirements satisfied). They found that straightening may even cause an increase of the unfavourable mode component in the distortion. This demonstrates that the costly and labourious fairing, by the use of the B-type tolerances, can even worsen the resistance of plates to uniaxial compression. Hence, from the point of view of compressive strength, the usefulness of straightening rectangular plates in order to satisfy the B-type tolerances is doubtful.

Therefore, the author proposes the following formulation of the permissible maximum amplitudes of IPD:

- for plates which the compressive strength has to be taken into account, the maximum amplitude of IPD should be measured using a gauge placed along the length of a plate (type A).

This, in spite of the fact that compressive strength of a plate depends more on the geometry than on the maximum amplitude of the IPD? Certainly not. The measurement of the maximum plate deflection over a short gauge length, equal to for instance plate breadth  $b$ , at any point along the length of a plate provides satisfactory representation of harmful wavelength distortions and localized dents having lengths in the range  $0.5b-1.2b$  without including the amplitude of less significant, longer wavelength distortions.

- for other purposes the maximum amplitude of IPD should be measured using a gauge placed along the breadth of a plate (type B).



The proposed formulation is, in the author's opinion, acceptable from the point of view of shipyard practice, because the measurement of the maximum amplitude of plating deflections is very simple, and is carried out according to the A- or the B-type specifications.

When tolerances are exceeded straightening is required. Straightening is performed mainly by heating. It is a highly labourious operation contributing considerably to the production costs of a ship. Regardless of economic aspects, straightening may exert a harmful influence on the strength properties of the material and in particular may reduce the resistance to brittle fracture [25]. In some cases, therefore, it may be preferable not to carry out any straightening.

In summarize: it is important to incorporate both types of tolerances in the specifications and to keep their values not too conservative, because straightening significantly increases production costs and can weaken the structure more than the deformation itself does.

#### 1.4. THE AIMS

To perform the statistical analyze of the A- and B-type maximum amplitudes of IPD calculated from the three dimensional measurements made by Kmiecik et al.

To find the statistical model of both amplitudes and to propose the new tolerances for allowable IPD.

The need for the model became apparent during the experimental work carried out by the author [26 and 27] on the compressive strength of repeatedly loaded plate panels. In that work a significant effect of the shape of IPD on the strength of the plate was found.

The present report includes the analysis, the model and the tolerance of the B-type amplitude of IPD.

Hereafter, if not otherwise stated, the amplitude of IPD is of the B-type.

## 1.5. MEASUREMENTS OF INITIAL PLATE DEFLECTIONS

Measurements of IPD by Kmiecik et al have been carried out in different shipyards and mainly on prefabricated sections of different kinds of ships. Use was made of specially designed equipment. Figure 5 shows schematically the measurements procedure and the main components of the equipment. Usually, plate deflections are measured and recorded with an interval of about 40 mm along the plate length for each of three or five positions of the frame on the plate breadth. Note here the important fact that one path of the plate deflection measurements is parallel to the long plate sides. Figure 2 shows an example of measured IPD.

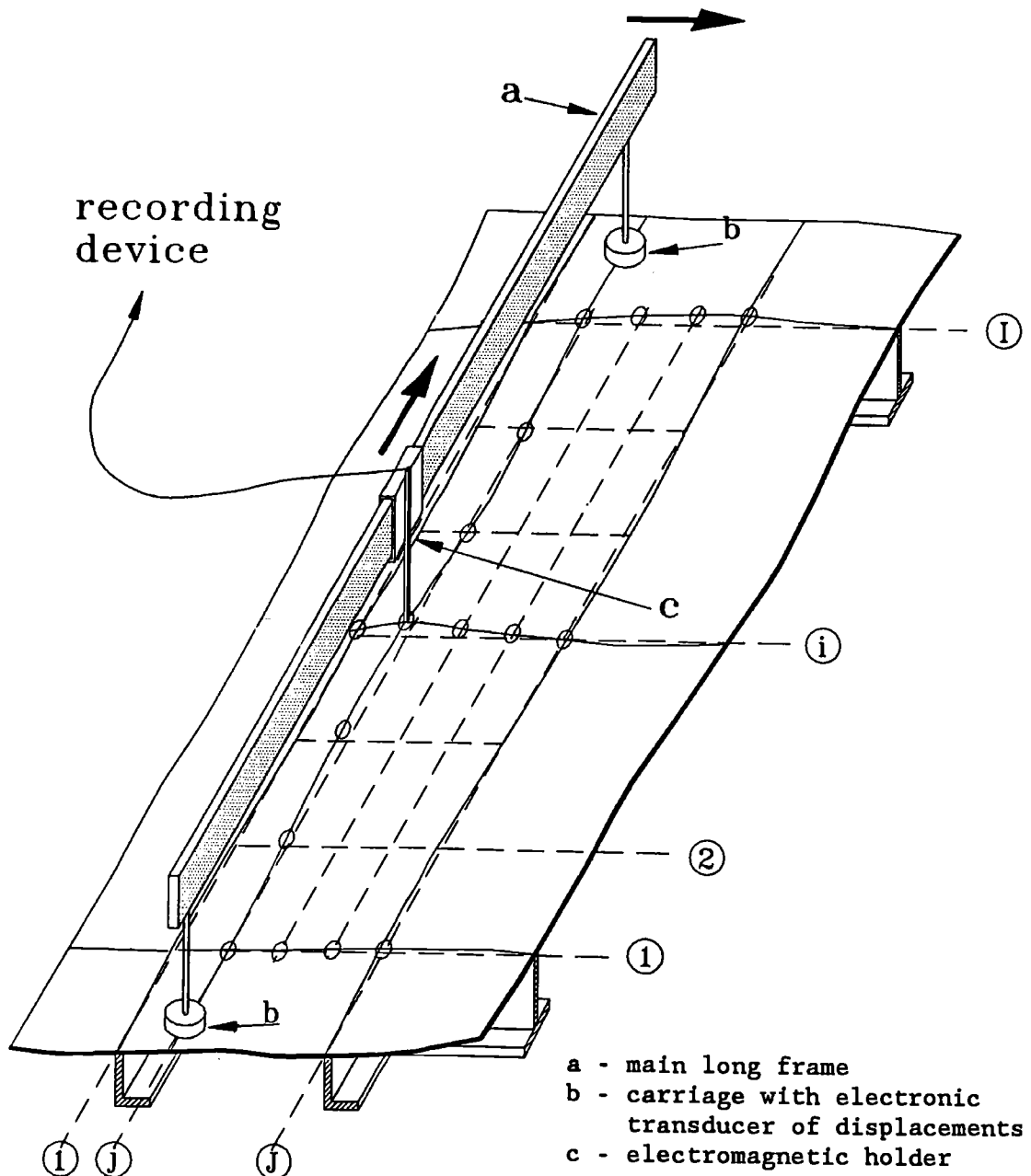


Fig. 5. Main device and procedure for IPD measurements used by Kmiecik.

## 1.6. MATHEMATICAL DESCRIPTION OF INITIAL PLATE DEFLECTIONS

The author proposes the following mathematical description of IPD, which represents the real initial out-of-plane deformation of a plate as the sum of the skewing and the bending components:

$$w_0(x,y) = w_0^a(x,y) + w_0^b(x,y)$$

The skewing component describes the uniform torsion of a plate together with its supporting stiffeners, and is mathematically represented by the hyperbolic paraboloid in the following form:

$$w_0^a(x,y) = \frac{x}{a} \frac{y}{b} s$$

- where  $s$  is the distance of a freely chosen plate corner from the plane defined by the three remaining corners,

The bending component describes the IPD in relation to the skewing component, and its mathematical representation depends on whether the IPD are localized or not.

The bending component of the global IPD is mathematically represented by one part of the whole Double Trigonometric Fourier Series (DTFS) depending on straightness of plate edges (stiffeners):

$$w_0^b(x,y) = w_0^F(x,y)$$

- all plate edges are straight:

$$w_0^F(x,y) = \sum_{m=1}^M \sum_{n=1}^N a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (1.6.1.)$$

- both short plate edges are straight:

$$w_0^F(x,y) = \sum_{m=1}^M \sum_{n=0}^N \lambda_{mn} b_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$$

- both long plate edges are straight:

$$w_0^F(x,y) = \sum_{m=0}^M \sum_{n=1}^N \lambda_{mn} c_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

- all plate edges are not straight:

$$w_0^F(x,y) = \sum_{m=0}^M \sum_{n=0}^N \lambda_{mn} d_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$$

- where:

$a, b$  - plate length and breadth

$0 \leq x \leq a$  and corresponds below to  $1 \leq i \leq I$

$0 \leq y \leq b$  and corresponds below to  $1 \leq j \leq J$

$\lambda_{00} = 1/4$ ,  $\lambda_{0n} = \lambda_{m0} = 1/2$ , in other cases  $\lambda_{mn} = 1$ ,

$a_{mn}, b_{mn}, c_{mn}, d_{mn}$  - coefficients of the double trigonometric Fourier series calculated from:

$$a_{mn} = \frac{4}{(I-1)(J-1)} \sum_{i=2}^{I-1} \sum_{j=2}^{J-1} W_0(i,j) \sin \frac{m\pi(i-1)}{I-1} \sin \frac{n\pi(j-1)}{J-1} \quad (1.6.2.)$$

- where  $W_0(i,j)$  represents a matrix of linearly transformed measured plate deflections to obtain the condition of straight short plate edges:  $w(1,j) = w(I,j) = 0$ ; the assumed straightness of long plate edges:  $w(i,1) = w(i,J) = 0$  is incorporated in the above equation by varying "i" index from 2 to I-1.

, and the remainder of the coefficients are defined in a similar way.

In the present work it is assumed that plate edges are straight and lie in one plane.

Note, that such a condition holds for the specimens tested in [27]). In such a case, only the sine-sine part of the Fourier series is needed to describe the IPD. Other descriptions are useful in stiffened plates analysis.

Further, the description of IPD with only the sine-sine part of DTFS makes it possible to examine their harmfulness, because the effect of particular IPD sine-sine modes on the plate strength is known [28] (see Section 4).

However, there are situations for which the DTFS coefficients may give a misleading and non-conservative representation of the IPD. The localized IPD [29] called dents are an example of such IPD. Therefore, the bending component of the localized IPD is mathematically represented by the Gauss bell surface, multiplied by a function  $g(x,y)$  in order to satisfy a condition of straight plate edges:

$$w_0^b(x,y) = w_0^d(x,y) = d g(x,y) e^{-\left[4 \frac{\lambda}{b} \left(\frac{2y}{b} - 1\right) \left(\frac{x-q}{b}\right)\right]^2}$$

where:  $\lambda$  - dent length  
 $d$  - dent amplitude  
 $q$  - position of a dent along the plate length:  $0 \leq q \leq a$

$$g(x,y) = \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$

The position  $q$  and dimensions of a depth  $d$  and  $\lambda$  may be calculated using the least squares procedure, i.e. minimizing the sum of squares between  $W_0(i,j)$  and  $w_0^d(x,y)$ . When local and global deformations are present, the sum of both bending components should be used.

## 2. MODEL OF MAXIMUM AMPLITUDE OF INITIAL PLATE DEFLECTIONS

### 2.1. GENERAL

The development of the empirical model involves the following steps:

- discussion of existing models,
- choice of important variables for an investigated phenomenon,
- postulation of a model,
- collection of data,
- fitting of the model, i.e., estimation of the model coefficients,
- evaluation of results.

Below, all steps are discussed in relation to the model for the B-type maximum amplitude of initial plate deflections (MIPD).

### 2.2. REVIEW OF EXISTING MODELS

In the models below:

- t - plate thickness
- b - plate length
- $t_w$  - web thickness of the long stiffener
- $\beta$  - plate slenderness  $\beta = b/t (\sigma_y/E)^{1/2}$
- $\sigma_y$  - yield stress
- E - Young's modulus
- w - maximum amplitude of initial plate deflections (MIPD),  
if not stated other it refers to the mean absolute value.

#### I. Faulkner's model [8]:

$$w/t = k \beta^2 t_w/t \quad \text{when} \quad t_w/t < 1$$

and:

$$w/t = k \beta^2 \quad \text{when} \quad t_w/t > 1$$

$$\text{where:} \quad \begin{array}{ll} k = 0.12 & 1 < \beta < 3 \\ k = 0.15 & \beta > 3 \end{array}$$

Method used: the least squares method of mean central values of the statistically grouped data set:  $\{ w/t ; \beta \}$ .

Note: in order to obtain the model in terms of the plate slenderness  $\beta$  the investigator has assumed the following values of the yield stress and Young's modulus:  $\sigma_y = 243$  [MPa],  $E = 204$  [GPa].

Hence, the model in terms of plate slenderness  $b/t$ , is:

$$\omega/t = 1.43 (b/t)^2 10^{-4} \quad (I)$$

provided that  $t_w/t \geq 1$  and  $30 < b/t < 90$ .

this form of Faulkner's model is hereafter compared with other models.

## II. Czujko and Kmiecik's first model [9]:

$$\omega/t = 0.00647 b/t + 0.0218 \quad 30 < b/t < 120 \quad (II)$$

Multiplying both sides of this model by plate thickness  $t$ , gives:

$$\omega = 0.00647 b + 0.0218 t$$

The last term on the right of the equation may be neglected, because it is only 2% of the thickness of the plate and also because this term is positive what means that the thicker the plate the bigger the amplitude of initial plate deflections. Thus, actually, the model suggests that the maximum plate deflection is only a function of plate breadth:

$$\omega = b / 155$$

Method used: the linear regression analysis (the least squares method) of the whole data set: (  $\omega/t$  ;  $b/t$  ).

## III. Czujko and Carlsen's model [14]:

$$\omega/t = 0.008 b/t - 0.13 \quad 30 < b/t < 120 \quad (III)$$

Method used: the linear regression analysis (the least squares method) of mean central values of the statistically grouped data set: (  $\omega/t$  ;  $b/t$  ).

The model defines also the maximum plate deflection, that will be not exceeded at a probability level of 97.7% :

$$\text{cumulative (97.7\%)} \omega/t = 0.016 b/t - 0.36 \quad (iii)$$

Method used: the linear regression analysis (the least squares method) of mean central values enlarged with two standard deviations of the statistically grouped data set: (  $\omega/t$  ;  $b/t$  ). The normal distribution of MIPD for a given value of plate slenderness was assumed.

IV. Antoniou's first model [17]:

$$w/t = k \beta^2 t_w/t \quad a/b > 2 ; 1 < \beta < 2.6$$

where:  $k = 0.091$  when  $t_w/t < 1$

$k = 0.0628$  when  $t_w/t > 1$

Note: in order to obtain the model in terms of plate slenderness  $\beta$  the investigator assumed the following values of the yield stress and Young's modulus:  $\sigma_y = 235$  [MPa],  $E = 206$  [GPa]. Hence, the model in terms of plate slenderness  $b/t$ , is:

$$w/t = k (b/t)^2$$

where:  $k = 7.18 \cdot 10^{-5}$  when  $t_w/t < 1$  (IVA)

$k = 10.4 \cdot 10^{-5}$  when  $t_w/t > 1$  (IVB)

this form of Antoniou's model is hereafter compared with other models.

What is disturbing in the model is the significant discontinuity (30%) in the predicted amplitude of maximum plate deflection when  $t_w/t=1$ .

The paper also gives the value of the  $k$  parameter calculated for all 1908 plates irrespective of parameters other than plate slenderness  $b/t$ :

$$k = 7.66 \cdot 10^{-5} \quad (\text{all measured plates}) \quad (\text{IVC})$$

Method used: the least squares method of the whole data set:  $(w/t; b/t)$ .

The model defines also the maximum plate deflection, that will be not exceeded at a probability level of 97.7% :

$$\text{cumulative (97.7\%)} \quad w/t = 0.014 b/t - 0.32 \quad t > 14 \text{ [mm]} \quad (\text{iva})$$

$$\text{cumulative (97.7\%)} \quad w/t = 0.018 b/t - 0.55 \quad t < 14 \text{ [mm]} \quad (\text{ivb})$$

Method used: see model (iii).

V. Jastrzebski's model [19]:

$$w/t = 0.008 b/t - 0.146 \quad (\text{V})$$

and:

$$\text{cumulative (97.7\%)} \quad w/t = 0.015 b/t - 0.298 \quad (\text{v})$$

Methods used: see models (III) and (iii) respectively.

VI. Antoniou's second model [18]:

$$w/t = 0.238 \beta - 0.177$$

Method used: see model (II).

Note: see IV.

In terms of the plate slenderness  $b/t$ , the model takes the following form:

$$w/t = 0.00805 b/t - 0.177 \quad (VI)$$

Figure 12 and Figure 13 compare the mean and the 97.7% cumulative values of MIPD obtained from above models, respectively.



### 2.3. DISCUSSION OF THE MODELS

The mean dimensionless absolute maximum amplitude of plate deflections  $w/t$  is defined in all models as a function of plate slenderness  $b/t$ . In addition, Faulkner's model and Antoniou's first model use different values of model parameters depending on the ratio of stiffener web thickness to plate thickness  $t_w/t$ . Some models use plate slenderness  $\beta$  in lieu of  $b/t$ . However, this extension of the models had been achieved artificially using an assumed, not a real, value, of the yield stress.

Three models define also the maximum plate deflection, that will be not exceeded at a probability level of 97.7 %, as a linear function of plate slenderness  $b/t$ .

There are two groups of models defining the mean value with respect to plate slenderness: linear and parabolic.

The author, will show that the above models are more the result of the assumptions made and of the chosen statistical method of analysis than the reflection of the investigated phenomenon.

In all cases, the parameters of postulated models have been estimated using the least squares method. To simplify the present discussion, it is assumed that a postulated model is in the form  $w/t = f(b/t)$ , and the distributions of  $w/t$  for given values of  $b/t$  are unknown.

The estimation of model parameters using the least squares method means nothing more than: the estimated set of parameters of the assumed model minimizes the sum of the squares of the deviations between the measured values  $w/t$  and the expected values  $w/t$  calculated from the model.

It means also that for freely chosen models which are in conflict with the physical nature of the investigated phenomenon, the least squares method will give such a set of model parameters.

Therefore, besides estimation of model parameters using the least squares method, additional analysis is required to test whether the model satisfactorily describes the sample of measured  $w/t$ . Such additional analysis is also necessary if one has to define the confidence interval for the estimated model parameters, or, especially, if one is interested not only in the mean values of  $w/t$  but also in the cumulative distributions of  $w/t$ .

The point is that such additional analyses using the least squares method require the normal distribution of the dependent random variable. As will be shown, this requirement is not satisfied in the problem addressed here. Therefore, special methods or tricks have to be used. Elsewhere in the text, one of those tricks, used by the author, is described.

In the literature referred to here, this aspect of the regression analysis using the least squares method is not pointed out. It seems that previous investigators have assumed the normal distribution of  $w/t$ . This may also be concluded from the way they have calculated the tolerance limit for allowable maximum plate deflection; namely, as mean plus two standard deviations, which is only valid in the case of normally distributed values.

## 2.4. CHOICE OF IMPORTANT VARIABLES

Generally, the chosen variables affecting the investigated phenomena should be mutually independent and should represent a similar significance level. From the point of view of practical use, their number should be limited and for the sake of convenience they ought to be dimensionless.

Note that the models discussed above do not use welding process parameters (welding speed) and geometrical parameters of the weld (throat thickness). For a given welded connection, these parameters are directly responsible for the amount of the heat input and the resulting shrinkage and distortions. Nevertheless, the author agrees with such an approach. Welding parameters are in fact incorporated in the models but in a hidden way. Namely, they are a function of joint geometry. If joint geometry parameters are chosen as variables, then in a statistical sense welding parameters are correlated to them and hence they can not be used as independent variables of a model.

Therefore, the model should be continuously re-examined for new data, in order to follow changes in the welding technology and requirements for weld dimensioning.

In this work, the above approach has been followed. From the models, it may be concluded that plate thickness and plate breadth have a dominating influence on the maximum plate deflection. The web thickness of long stiffeners and yield stress has less influence. Hence, it can be assumed that the maximum plate deflection is a function only of plate thickness and plate breadth:

$$w = f(b, t)$$

For the sake of convenience, dimensionless variables are used as has been done by other researchers:

$$w/t = f(b/t)$$

It follows that the dimensionless maximum amplitude of initial plate deflections is a function of the plate slenderness, or equivalently that the maximum initial plate deflection is a part of the plate thickness, and the part is a function of the plate slenderness:

$$w = f(b/t) t$$

TABLE 4

The slenderness  $b/t$  and the dimensionless maximum B-type deflection amplitude for 411 different ship's plates

$b/t$	$w/t$	$b/t$	$w/t$	$b/t$	$w/t$	$b/t$	$w/t$	$b/t$	$w/t$	$b/t$	$w/t$
48.7	-0.165	70.6	0.178	58.8	-0.213	51.5	-0.272	41.2	0.216	41.2	-0.102
48.7	0.314	70.6	-0.255	58.8	-0.357	51.5	-0.194	41.2	0.121	41.2	-0.150
48.7	0.267	70.6	-0.255	58.8	-0.399	51.5	-0.204	41.2	-0.238	41.2	-0.203
48.7	0.141	70.6	0.153	58.8	-0.399	51.5	0.126	41.2	-0.131	41.2	-0.105
48.7	-0.092	70.6	-0.255	58.8	-0.272	51.5	-0.378	41.2	-0.082	41.2	-0.214
48.7	0.091	70.6	-0.255	58.8	0.136	51.5	-0.126	41.2	-0.403	41.2	-0.105
48.7	-0.052	70.6	-0.433	58.8	-0.094	51.5	-0.165	41.2	-0.339	41.2	-0.200
48.7	-0.143	70.6	-0.187	52.6	-0.399	51.5	-0.272	41.2	-0.138	41.2	-0.199
48.7	0.108	70.6	-0.366	51.5	-0.242	51.5	0.175	41.2	-0.166	41.2	-0.152
48.7	0.077	70.6	0.655	51.5	-0.223	51.5	-0.359	25.0	-0.056	41.2	-0.266
48.7	0.109	70.6	-0.161	52.6	-0.380	51.5	-0.194	50.0	-0.271	41.2	-0.058
48.7	0.116	70.6	0.417	52.6	-0.181	51.5	0.204	50.0	-0.223	41.2	-0.180
48.7	0.105	48.8	0.160	52.6	-0.370	51.5	-0.339	41.2	-0.327	41.2	-0.204
30.0	0.142	70.6	0.425	52.5	-0.219	51.5	-0.136	25.0	-0.098	41.2	-0.135
33.3	-0.203	70.6	0.561	52.6	-0.181	51.5	-0.078	41.2	0.160	41.2	-0.299
66.7	-0.198	70.6	-0.221	52.6	-0.162	70.4	-0.561	41.2	-0.109	41.2	-0.167
50.0	0.282	70.6	-0.502	52.6	-0.133	51.5	0.175	41.2	-0.116	41.2	-0.133
65.5	0.569	70.6	-0.476	52.6	0.162	70.6	-0.427	41.2	0.198	41.2	-0.173
67.3	0.511	70.6	-0.230	52.6	0.798	70.6	-0.219	41.2	-0.130	41.2	-0.213
68.2	0.542	70.6	-0.578	52.6	-0.181	70.6	-0.201	41.2	-0.129	41.2	-0.184
67.3	0.642	70.6	0.238	52.6	-0.294	70.6	-0.927	50.0	-0.167	41.2	-0.174
67.3	0.536	70.6	-0.638	52.6	-0.228	70.6	-0.900	50.0	-0.251	41.2	-0.213
68.2	0.594	70.6	-0.620	52.6	-0.171	70.6	-0.732	50.0	-0.217	41.2	0.024
63.6	-0.401	70.6	-0.544	52.6	-0.237	70.6	-1.038	50.0	-0.153	41.2	-0.046
30.0	-0.117	70.6	-0.340	52.6	-0.285	70.6	-0.479	50.0	-0.214	41.2	-0.116
30.0	-0.135	70.6	-0.238	52.6	-0.133	70.6	0.616	50.0	-0.169	41.2	-0.284
30.0	-0.131	70.6	-0.527	52.6	-0.190	70.6	-0.624	50.0	-0.179	41.2	-0.092
30.0	-0.116	70.6	-0.264	52.6	-0.285	70.6	-0.457	50.0	-0.164	41.2	-0.083
48.7	-0.125	70.6	-0.349	37.1	-0.242	70.6	-0.571	41.2	-0.057	41.2	-0.296
48.7	-0.135	70.6	0.213	37.6	-0.187	70.6	2.350	41.2	-0.113	41.2	-0.091
48.7	0.261	70.6	0.181	37.1	0.187	70.6	-0.541	41.2	-0.157	41.2	-0.113
48.7	0.117	70.6	-0.612	48.8	-0.172	70.6	-0.575	41.2	-0.082	41.2	-0.127
68.2	0.746	70.6	-0.476	48.8	-0.431	70.6	2.369	41.2	0.052	41.2	-0.202
30.0	-0.121	70.6	-0.306	70.6	-0.620	70.6	0.744	41.2	0.117	41.2	-0.084
60.8	-0.350	70.6	-0.417	70.6	-0.629	70.6	-0.421	41.2	0.075	41.2	-0.141
37.1	-0.223	70.6	-0.306	70.6	0.442	70.6	1.349	41.2	-0.162	41.2	-0.138
37.1	0.186	70.6	-0.383	70.6	0.349	70.6	-0.597	41.2	-0.135	41.2	-0.322
37.6	0.149	70.6	-0.280	51.5	-0.213	70.6	-0.847	41.2	-0.131	41.2	-0.122
37.6	-0.242	70.6	-0.391	58.8	-0.391	20.0	-0.129	41.2	-0.124	41.2	-0.113
48.8	-0.123	70.6	-0.433	52.6	-0.171	20.0	0.173	41.2	-0.157	41.2	-0.099
48.8	0.086	70.6	-0.425	70.6	-0.952	20.0	0.192	41.2	-0.267	41.2	-0.140
48.8	-0.209	51.5	-0.213	70.6	-0.221	20.0	0.153	41.2	-0.192	50.0	-0.249
48.8	-0.111	51.5	-0.242	51.5	-0.175	20.0	0.139	41.2	-0.193	41.2	-0.105
48.8	-0.381	51.5	-0.194	51.5	-0.088	20.0	0.108	41.2	-0.209	41.2	-0.073
48.8	-0.098	51.5	-0.068	51.5	-0.155	20.0	0.191	41.2	-0.132	41.2	-0.127
48.8	0.148	51.5	-0.369	52.6	-0.124	20.0	0.207	41.2	-0.215	41.2	-0.152
48.8	-0.332	51.5	-0.262	37.6	0.112	20.0	0.235	41.2	-0.235	41.2	-0.265
48.8	-0.111	51.5	-0.107	52.6	-0.313	20.0	-0.127	41.2	-0.181	41.2	-0.081
48.8	-0.381	51.5	-0.184	52.6	-0.409	20.0	0.208	41.2	-0.185	41.2	-0.216
48.8	-0.172	51.5	-0.194	52.6	-0.352	20.0	0.134	41.2	-0.162	41.2	-0.135
48.8	0.098	51.5	-0.242	52.6	-0.370	20.0	-0.149	41.2	-0.349	41.2	-0.132
48.8	-0.320	51.5	-0.252	52.6	-0.181	20.0	-0.170	41.2	-0.199	51.5	-0.437
48.8	-0.111	51.5	-0.252	52.6	0.152	20.0	0.159	41.2	-0.130	51.5	-0.184
48.8	-0.111	51.5	-0.194	52.6	-0.095	80.0	-0.386	41.2	-0.201	51.5	-0.175
48.8	-0.086	51.5	-0.213	52.6	-0.162	80.0	-0.780	41.2	-0.141	51.5	-0.145
48.8	0.180	58.8	-0.272	52.6	-0.352	80.0	0.467	41.2	-0.147	51.5	-0.087
48.8	-0.184	58.8	-0.247	52.6	-0.408	113.	-1.445	41.2	-0.112	58.8	0.213
48.8	0.271	58.8	-0.289	51.5	-0.272	113.	0.744	41.2	-0.158	58.8	0.196
70.4	0.170	51.5	0.184	51.5	-0.155	113.	1.762	41.2	-0.251	58.8	-0.493
70.4	-0.561	51.5	-0.107	51.5	-0.175	113.	-1.307	41.2	-0.152	58.8	-0.510
70.6	-0.629	51.5	0.242	51.5	-0.223	41.2	-0.093	41.2	-0.135	58.8	-0.127
70.6	-0.280	58.8	-0.111	51.5	0.184	41.2	-0.182	41.2	-0.165	70.6	-0.289
70.6	-0.442	58.8	-0.247	70.6	-0.247	41.2	0.198	50.0	0.182	70.6	-0.272
70.6	-0.323	58.8	0.161	70.6	-0.323	41.2	0.123	50.0	-0.193	70.6	-0.357
70.6	-0.272	58.8	0.161	51.5	-0.204	41.2	-0.179	50.0	-0.189	70.6	-0.306
70.6	-0.153	58.8	-0.230	51.5	-0.155	41.2	-0.076	50.0	-0.203	70.6	-0.553
70.6	-0.247	58.8	0.153	51.5	-0.155	41.2	-0.193	50.0	-0.201		
70.6	-0.187	58.8	-0.178	51.5	-0.107	41.2	0.070	50.0	-0.257		
70.6	-0.204	58.8	0.170	51.5	0.184	41.2	-0.076	50.0	-0.253		

## 2.5. POSTULATION OF THE MODEL

To find the functional relation, the required data were selected from the data base and were analyzed by the interactive computer programme GRAMIR developed by the author. The programme uses statistical methods described in [30]. The figures presented below form the graphic output of the programme.

Table 4 contains a list of data on 411 different ships's plates. The slenderness  $b/t$  and the dimensionless maximum B-type deflection is given for each plate.

Table 4 shows that 22% of the plates have a negative maximum plate deflection. The analysis, not included here, has shown that the further model of absolute maximum deflection obtained hereafter falls between confidence limits of models obtained respectively for exclusively positive and exclusively negative maximum deflections. Hence, further analysis has been made irrespective of the sign of deflection. (i.e. the Dimensionless Absolute Maximum of Initial Plate Deflections of a plate is further used, DAMIPD-is the used abbreviation).

Note, that from the point of view of the analysis of a single plate the sign of maximum deflection is not important, but from the point of view of the analysis of stiffened panels the sign of maximum deflection plays an important part [31].

Thus, the following functional relationship is sought:

$$\text{mean} ( |w|/t ) = f( b/t ) \quad (2.5.1.)$$

Further, DAMIPD is treated as dependent random variable and plate slenderness is assumed to be an independent deterministic variable.

The analysis is done in the following way.

First the scatter (Figure 6) and the histogram (Figure 7) of DAMIPD and plate slenderness are visually analyzed. The following conclusions have been derived:

- DAMIPD may be assumed constant for stocky plates;

This restricts the validity range of the future model to plates with  $b/t > 40$ . For plates with lower slenderness, the same distribution and mean value of DAMIPD are assumed as the model will determine for plates with  $b/t=40$ .

- the relation between DAMIPD and plate slenderness has a parabolic character for  $b/t > 40$ ;

This means that a parabolic model should be postulated rather than a linear one.

- the more slender the plate the higher the scatter of DAMIPD;

dimensionless absolute maximum deflection  
 $|w|/t$

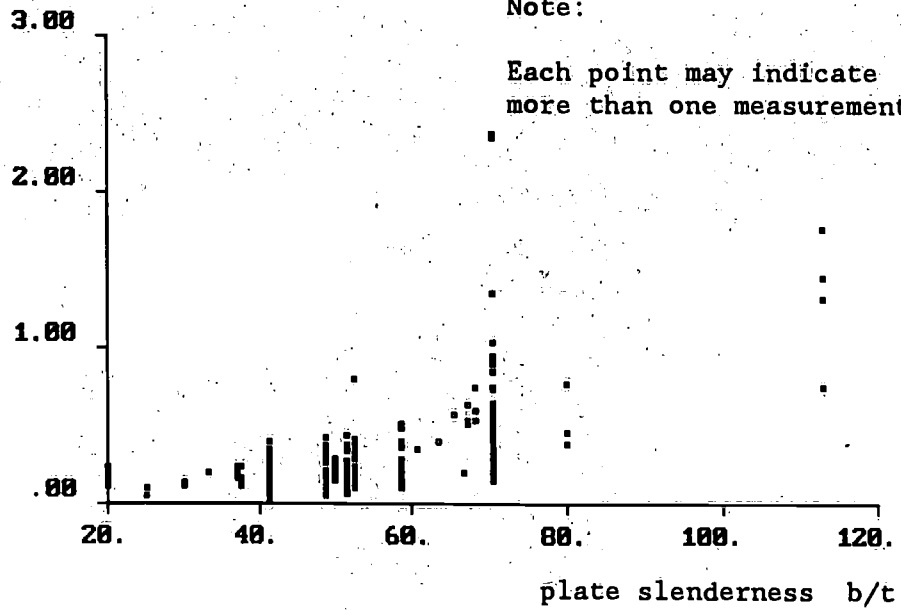


Fig. 6. Scatter of dimensionless absolute maximum initial plate deflection (DAMIPD).

dimensionless absolute maximum deflection  
 $|w|/t$

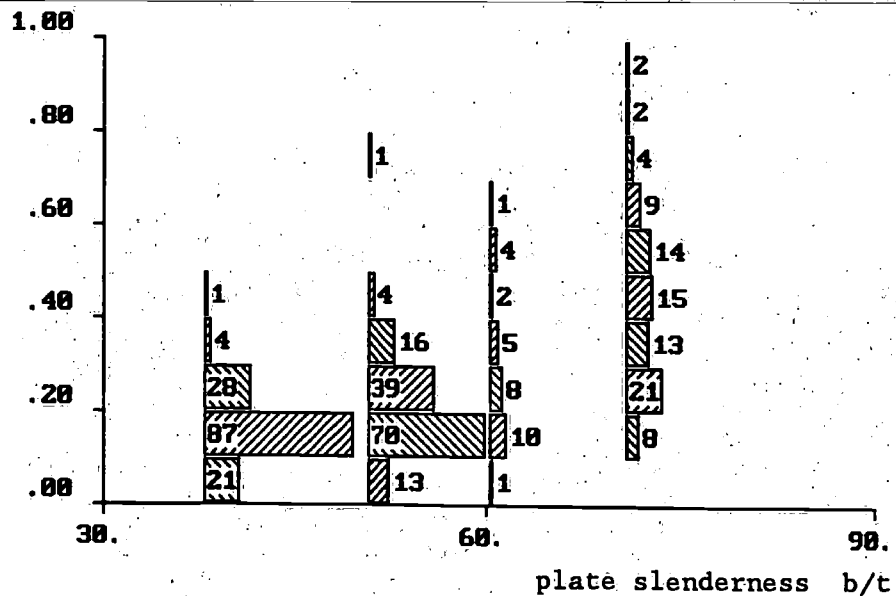


Fig. 7. Histograms of dimensionless absolute maximum initial plate deflection (DAMIPD).

- there is a small amount of data on very slender plates  $b/t > 80$ ;

Obviously, the small amount of data on slender plates is unsatisfactory for the purpose of statistical analysis. Additional analysis has shown that the model of DAMIPD derived hereafter is not affected by whether plates with slenderness  $b/t > 80$  are excluded from the data sample or not. This indicates that the small amount of data on slender plates is representative, and that a tendency in the model is strongly enough represented by remained data. Hence, in the analysis presented hereafter all available data  $b/t > 40$  are used. However, the validity of the model can be only guaranteed for plates with  $40 < b/t < 80$ .

- the distribution of DAMIPD for given plate slenderness does not follow the normal distribution;

- this means that the additional analyses using the least squares method can not be performed. A special method will have to be used in order to satisfy the requirement for normal distribution of DAMIPD.

A useful manner of satisfying the requirement for the normal distribution of a dependent random variable is to transform it in such a way that its distribution becomes normal. Further, it is also reasonable to transform the independent variable in such a way that a linear model can be postulated. If both transformations exist, then a statistical analysis using the least squares method may be performed on new transformed data. If the linear model is not rejected, the results are transformed to original coordinates. In the opposite case a new transformation has to be found. If it is impossible to find such a transformation, another statistical method should be applied.

Himmelblau in [30] presents guidelines to finding such transformations. The author has followed these guidelines and has found that the normal logarithmic transformation of both values satisfies the requirements.

Figure 8 presents the logarithmically transformed data of Figure 6 ( $b/t > 40$ ). From this figure the following conclusions can be derived:

- in new coordinates the linear model may be postulated:

$$\text{mean} ( \ln ( |w|/t ) ) = A \ln ( b/t ) + B \quad (2.5.2.)$$

where A and B are constants of which the values will have to be estimated;

- it may be assumed that the variance of LDAMIPD is a constant, independent of  $\ln(b/t)$ . (LDAMIPD = natural Logarithm of DAMIPD ).

The next step is the grouping of the data. There are several reasons for doing this:

- the regression analysis of grouped data is less affected by the distribution of the independent value,

normal logarithm of dimensionless absolute maximum deflection  
 $\ln(|w|/t)$

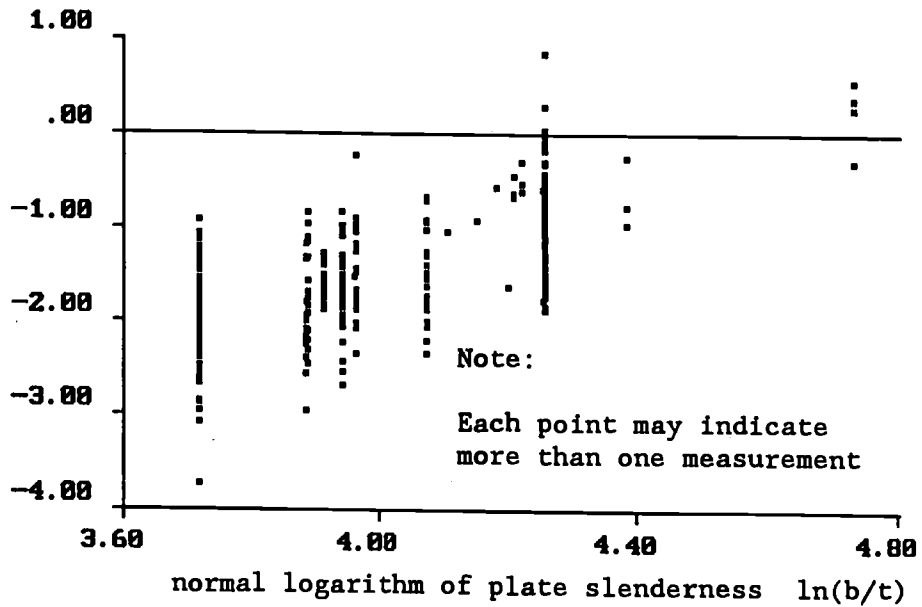


Fig. 8. Scatter of normal logarithm of dimensionless absolute maximum initial plate deflection (LDAMIPD).

normal logarithm of dimensionless absolute maximum deflection  
 $\ln(|w|/t)$

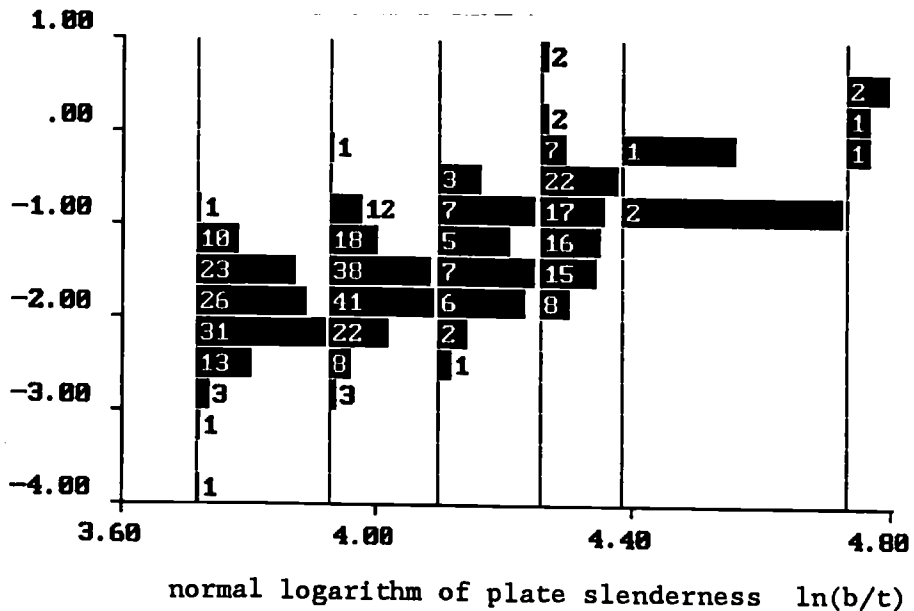


Fig. 9. Histograms of normal logarithm of dimensionless absolute maximum initial plate deflection (LDAMIPD).

- grouping gives additional information, used, for instance, to test the hypothesis that the model represents the data satisfactorily or to calculate the confidence region. The next step is the grouping of the data. There are several reasons for doing this:

Grouping has one disadvantage - it introduces new factors which may affect the further analysis. The author has grouped the data into data sets using the following criteria:

- data should be equally distributed between data sets,
- the distance between successive means of data sets should be equal.

Figure 9 shows histograms of data sets. The horizontal position of each histogram corresponds to the mean value of the data set.

It is possible now to test whether data within each set is normally distributed. This has been done by means of the Test of Goodness of Fit described in [30], and, of course, only for the first four histograms. It is found that normal distribution is representative of all tested histograms.

Thus, the requirement of normal distribution of the dependent variable is now satisfied and the analysis may proceed. Before the regression calculation is carried out it is necessary to test the hypothesis of constant variance of LDAMIPD. Because the normal distribution of LDAMIPD has been proved, it is possible to use Bartlett's test. This test has proved the hypothesis to be correct.

## 2.6. FITTING OF THE MODEL

Figure 10 shows the means and the standard deviations of the data sets and the results of the regression analysis:

- regression line - linear relation between mean LDAMIPD and  $\ln(b/t)$ , the estimated values of model coefficients are:

$$A = 1.98$$

$$B = - 9.37$$

- confidence region, it can be seen that all means lie inside the confidence interval;

- the constant standard deviation of LDAMIPD is found to be:

$$\sigma = 0.492$$

- the estimated normal probability distribution of LDAMIPD for each data set.



normal logarithm of dimensionless absolute maximum deflection  
 $\ln (|w|/t)$

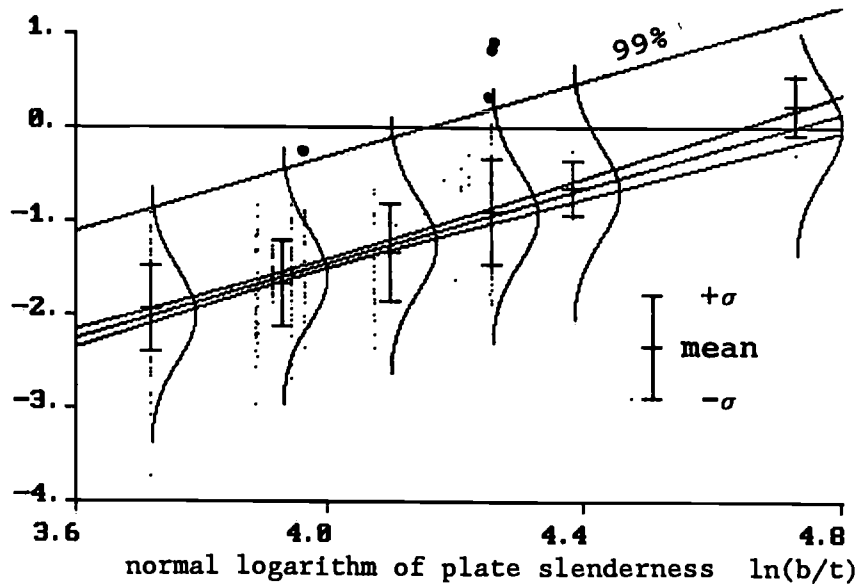


Fig. 10. Results of regression analysis.

To test whether the linear model satisfactorily represents the data, the F-test has been performed with respect to the variance ratio:

$$\frac{s_r^2}{s_e^2} = 1.99 < 2.37 = F_{1-\alpha}(4; 373)$$

- where:

$s_r^2$  - variance of means of data sets about the regression line  
 ( 4 degrees of freedom in the case considered here ),

$s_e^2$  - variance of data within all sets  
 ( 373 degrees of freedom in the case considered here ),

$\alpha$  - the significance level.

,and as it can be seen the model is not rejected. This does not mean that the model is the best one possible. However, the model presented here is the best of models of the assumed type tested by the author 2.5.1.

Note that in all tests performed, the significance level  $\alpha = 0.05$  was used.

## 2.7. EVALUATION OF THE MODEL

From the point of view of the practical use of the model it is more important to know the maximum deflection that will be not exceeded at a certain probability level than to know the mean value. Therefore the author has decided to incorporate this information in the model.

Because of the normal distribution of LDAMIPD, its cumulative value for a given probability level and  $\ln(b/t)$ , is equal to:

$$\ln ( |\omega|/t ) = A \ln(b/t) + B + u \sigma \quad (2.7.1.)$$

where  $u$  is the standard normal variable for a given cumulative probability level (see Table 5).

The above equation represents a line parallel to the regression line and is hereafter called the cumulative regression line.

Figure 10 shows the 99% cumulative regression line ( $u = 2.33$ ). It can be seen from the figure that four measurements (1% of sample size  $379 \approx 4$ ) lie indeed above the 99% cumulative distribution line. This is one of the tests which confirm the correctness of the model.

Before the transformation of results into original coordinates, it is reasonable to test the hypothesis that  $A = 2$ . Not going into detail and referring to [30] the hypothesis is accepted and the new values are:

$$A = 2.00$$

$$B = - 9.45$$

The linear model 2.7.1 in logarithmic coordinates corresponds to the parabolic model in original coordinates:

$$|\omega|/t = k (b/t)^A$$

$$\text{where: } k = e^{B+u\sigma}$$

Substituting for  $A$ ,  $B$  and  $\sigma$  the estimated values, the model takes the following final form:

$$|\omega|/t = k (b/t)^2 \quad (2.7.2.)$$

where the value of parameter  $k$  should be determined on the basis of Table 5. The proposed validity range of the model is  $40 < b/t < 120$ .

Figure 11 presents the obtained model of DAMIPD. The equation 2.7.2 is plotted for different values of parameter  $k$  from Table 5.

**TABLE 5**

Values of parameters k and C

Maximum initial deflection	Standard normal variable	Corresponding significance level	Value of parameter k x 10 <sup>-5</sup>	Value of parameter C
$ w /t$	u	$\alpha$	k	C
expected	- $\sigma$ = - 0.492	—	6.18	0.345
mean				
50%	0	0.5	7.87	0.440
standard tolerance				
95 %	1.6450	0.05	17.7	1.000
limit tolerance				
99.75 %	2.8075	0.0025	31.3	1.750
other cumulative values:				
75 %	0.6745	0.25	11.0	0.615
90 %	1.2815	0.1	14.7	0.825
97.5 %	1.9600	0.025	20.6	1.155
99 %	2.3265	0.01	24.7	1.385
one-sided cumulative values corresponding to <u>mean + u <math>\sigma</math></u> in normal probability distribution:				
84.134 %	1	0.15876	12.9	0.730
97.725 %	2	0.02275	21.1	1.190
99.865 %	3	0.00135	34.4	1.950

The normal distribution of LDAMIPD becomes the log-normal distribution of DAMIPD in original coordinates:

$$p \left( \frac{w}{t} \right) = \frac{1}{\frac{w}{t} \sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2} \left( \ln \frac{w}{t} - B - A \ln \frac{b}{t} \right)^2} \quad (2.7.3.)$$

This equation defines the probability of occurrence of a given value of maximum plate deflection in relation to plate slenderness.

Figure 11 shows distributions of DAMIPD for four typical values of plate slenderness. As was concluded from the data scatter, the model shows a significant growth in the spreading of the maximum plate deflection with increasing the plate slenderness.

dimensionless absolute maximum B-type deflection amplitude

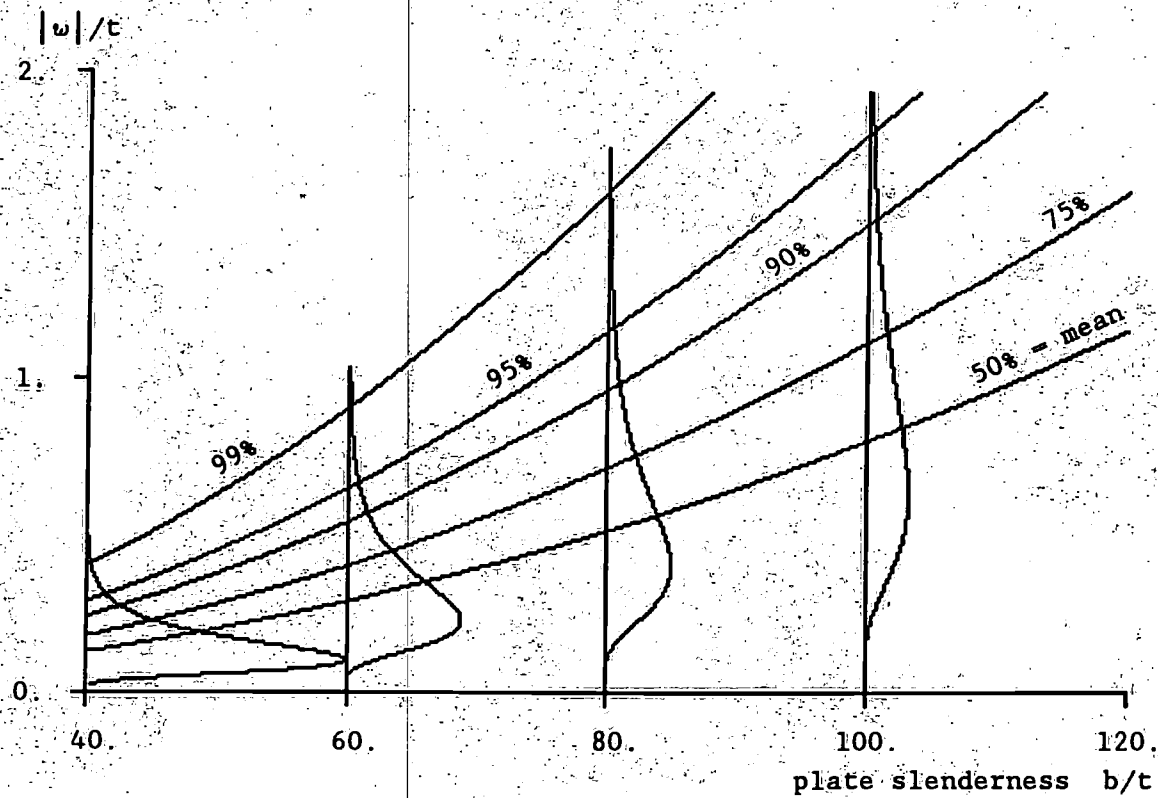


Fig. 11. Model of dimensionless absolute maximum B-type amplitude of IPD.

## 2.8. COMPARISON WITH OTHER MODELS

Figure 12 compares the mean values of MIPD as obtained by the author and by other investigators. From this figure, the following conclusions have been derived:

- there is very good agreement between the mean value of MIPD defined by the new model and Antoniou's IVC model. Both models are obtained by analyses of whole but different data samples, irrespective of parameters other than plate slenderness. This agreement is more expected than surprising, because only estimation of the mean value using the least squares method is independent of the assumption made as to the kind of distribution of random value;

- linear models overestimate or underestimate the mean value of MIPD, depending on whether the value of plate slenderness is lower or greater than  $b/t = 80$ .

dimensionless absolute maximum deflection  
 $|w|/t$

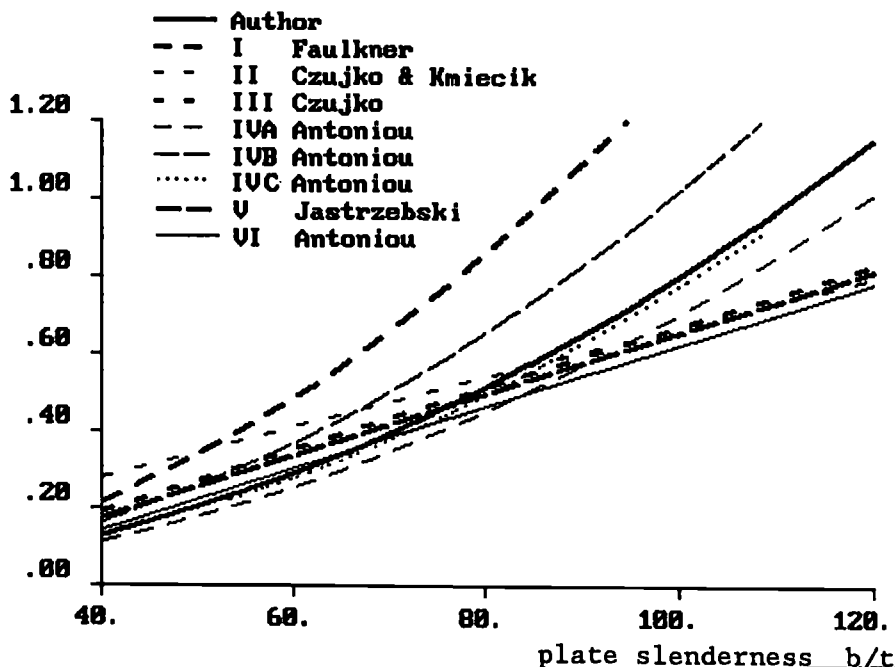


Fig. 12. Mean maximum plate deflection obtained from different models.

dimensionless absolute maximum deflection  
 $|\omega|/t$

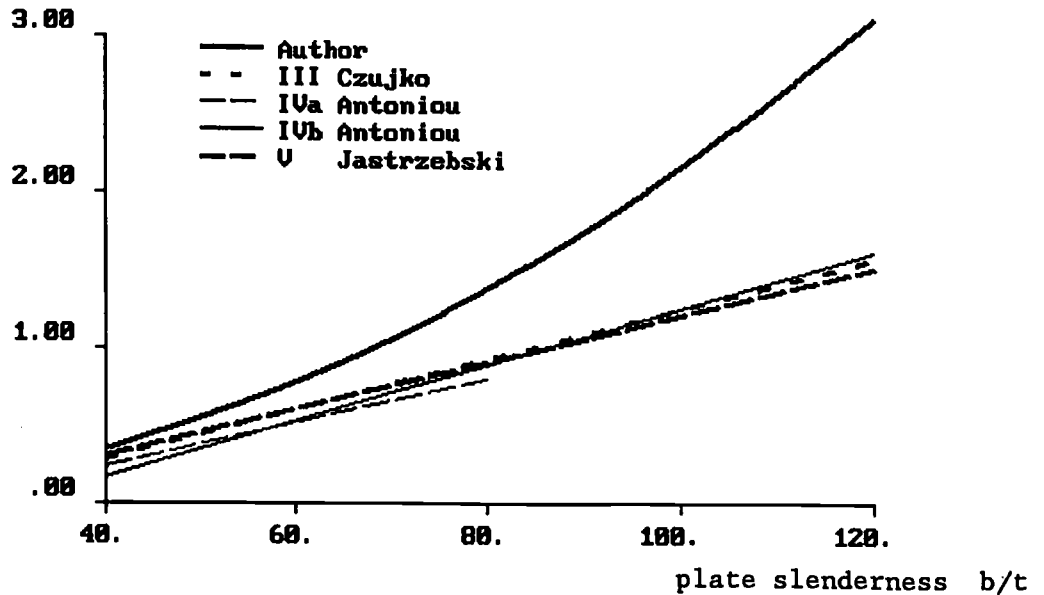


Fig. 13. Maximum plate deflection which will be not exceeded with the probability of 97.7% obtained from different models.

Figure 13 compares the 97.7% cumulative values of MIPD for one single plate obtained by the author and by other investigators. From this figure the following conclusion has been derived:

- the new model, in comparison with other models, defines considerably higher values of the 97.7% cumulative MIPD. This difference is caused by the fact that previous investigators, when defining the tolerance limit, assumed the normal distribution of maximum plate deflection, which is not in agreement with the real situation.

This conclusion brings the validity of the limit tolerances for allowable maximum plate deflections developed using these models into question.

### 3. PROPOSAL OF SPECIFICATION OF PERMISSIBLE PLATE DEFLECTIONS

#### 3.1. GENERAL

There are different criteria for assessing the tolerance limits for distortions:

- the practical criterion

the practical criterion is derived from the statistical analysis of distortions measured on real structures; the tolerance limit is the value of a distortion for which the probability of exceedance is equal to a certain level, say 5% for the standard value and 0.25% for the limit value.

- the deterministic criterion

the deterministic criterion is derived from the theoretical or the experimental investigation which provides the relation between the value of a distortion and the strength of the structural element; the tolerance limit is the value of a distortion which in relation to an ideal configuration causes a certain reduction of the strength, say 10% for the standard value and 20% for the limit value.

- the reliability criterion

the reliability criterion is derived from the reliability analysis; the tolerance limit is the value of a distortion which gives a certain level of the probability of failure of a structural element, say once per 100 years; note that the reliability analysis requires knowledge of the distribution of the loads [32].

- the cost criterion

the cost criterion is derived from the cost analysis; the tolerance limit is the value of an allowable distortion which minimizes total fabrication costs; such an optimum tolerance exists because tolerances which are too conservative increase the straightening costs and decrease the cost of other operations, such as the assembly and installation of equipment, whereas tolerances which are too liberal bring about the opposite effect.

Hereafter, the practical criterion will be applied in order to assess the new tolerance limit for the maximum B-type amplitude of IPD using the model derived in Section 2.7.

### 3.2. PROPOSAL

Multiplying relation 2.7.2 by the plate thickness  $t$  gives:

$$w = \frac{b}{100} \frac{b/t}{56} C \quad (3.2.1.)$$

where  $C$  is a parameter for which values are given in Table 5.

Note that the number 56 is easy to remember because it corresponds to the value of the slenderness  $b/t$  for a simply supported square plate, made of mild steel, for which the critical Euler's stress is equal to the yield stress limit.

Inserting in equation 3.2.1 the appropriate values of  $C$ , gives the following tolerance limits for the maximum B-type amplitude of initial plate deflections:

the standard value (95%):

$$q = \frac{b}{100} \frac{b/t}{56} \quad (3.2.2.)$$

the limit value (99.75%):

$$q = \frac{b}{56} \frac{b/t}{56} \quad (3.2.3.)$$

Of course, using Table 5 the tolerance levels for other probability levels can be assessed.

For plates with a given breadth, the new tolerances compared to the tolerances discussed in Section 1.2, allow for higher distortions when the plate is thinner.

Table 3 shows that for the example of the plate, the new tolerances allow for 50% higher distortions in comparison to the existing standards.



## 4. MODEL OF THE SHAPE OF INITIAL PLATE DEFLECTIONS

A model of the shape of initial plate deflections is important when the in-plane strength of a plate is being considered.

The shape of initial plate deflections is represented by the double Fourier series 1.6.1. The reason for this is obvious, namely, coefficients  $a_{mn}$  of the Fourier series represent the amplitudes of adequate modes with  $m$  half-waves in the longitudinal direction and  $n$  half-waves in the transverse direction. The existence of only one of such modes, which coincides with the buckling mode, causes the decrease of the plate strength. Thus, the coefficient corresponding to that mode may be used as a measure of the harmfulness of the real complicated shapes of IPD.

The buckling mode of longitudinally in-plane compressed simply supported plate has one half-wave in the transverse direction ( $n=1$ ) and  $m$  half-waves in the longitudinal direction, depending on the aspect ratio  $a/b$ :

Therefore, it is reasonable to restrict the model of the shape to the model of the  $a_{h1}$  coefficients.

No data which includes the  $a_{h1}$  coefficients was available to the author to make his own analysis. Thus, some results of other investigators are presented here. As was discussed in the introduction to the present work, there are two such sources of information known to the author: the data base of Kmiecik et al and Antoniou's second work [18].

The  $a_{mn}$  coefficients included in Kmiecik's data base have not yet been fully analyzed. Reference [19], however, presents some temporary results in the form of  $|a_{h1}/w|$  distributions for different plate aspect ratios (Figure 14). This form of data presentation was introduced by Czujko and Carlsen in [14] to specify the plate deflection tolerances in terms of the harmful coefficients  $a_{h1}$ . One serious disadvantage of this form of data representation is that it gives no information about the value of the harmful coefficients.

TABLE 6

Harmful DTFS coefficients

aspect ratio range	$m$	$a_{h1}$ - harmful $a_{mn}$ coefficient
$1.00 < a/b < 1.41$	1	$a_{11}$
$1.41 < a/b < 2.45$	2	$a_{21}$
$2.45 < a/b < 3.46$	3	$a_{31}$
$3.46 < a/b < 4.47$	4	$a_{41}$

Furthermore, in light of the findings of the previous section, there is a need to examine the validity of these tolerances because Figure 14 clearly shows the non-normal distribution of  $|a_{h1}/w|$ . The same holds for the tolerances proposed by Antoniou.

Antoniou's paper here referred to presents cumulative distributions of the above-mentioned  $|a_{h1}/w|$  ratios and also cumulative distributions of  $|a_{h1}|/t$ . Table 7 shows selected information on these two distributions. Antoniou has also made an attempt to express the  $a_{h1}$  coefficients as linear functions of the geometrical parameters. But, unfortunately, results of the regression analysis were unacceptably influenced by the statistical distribution of the data. However, he found a predominant influence of the plate slenderness  $\beta$ .

The concept of using the  $a_{h1}$  coefficients as a measure of the harmfulness of the shapes of IPD, however, although important from the point of view of the explanation of the effect of the shape of IPD on the compressive strength, has several important disadvantages:

- there are situations for which the DTFS coefficients may give a misleading and non-conservative representation of the IPD. The localized IPD (dent) is an example of such an IPD.
- numerical and experimental research shows that not only the  $a_{h1}$  coefficients listed in Table 6 but also coefficients representing wave distortions with lengths in the range  $0.5b-1.2b$  have a harmful effect on the compressive strength of rectangular plates.
- a check of IPD in regard to possible tolerances based on DTFS coefficients would be more time-consuming and would require extensive measurements of IPD using special equipment; this is difficult to accept from the point of view of shipyard practice.
- uniaxial compressive strength is the only factor which requires the investigation of the geometry of IPD.

Finally, it may be concluded that the concept of using the  $a_{h1}$  coefficients as a measure of the harmfulness of the real, complicated shapes of IPD should not be recommended because of the important disadvantages presented above and the low statistical correlation discussed in previous paragraphs.

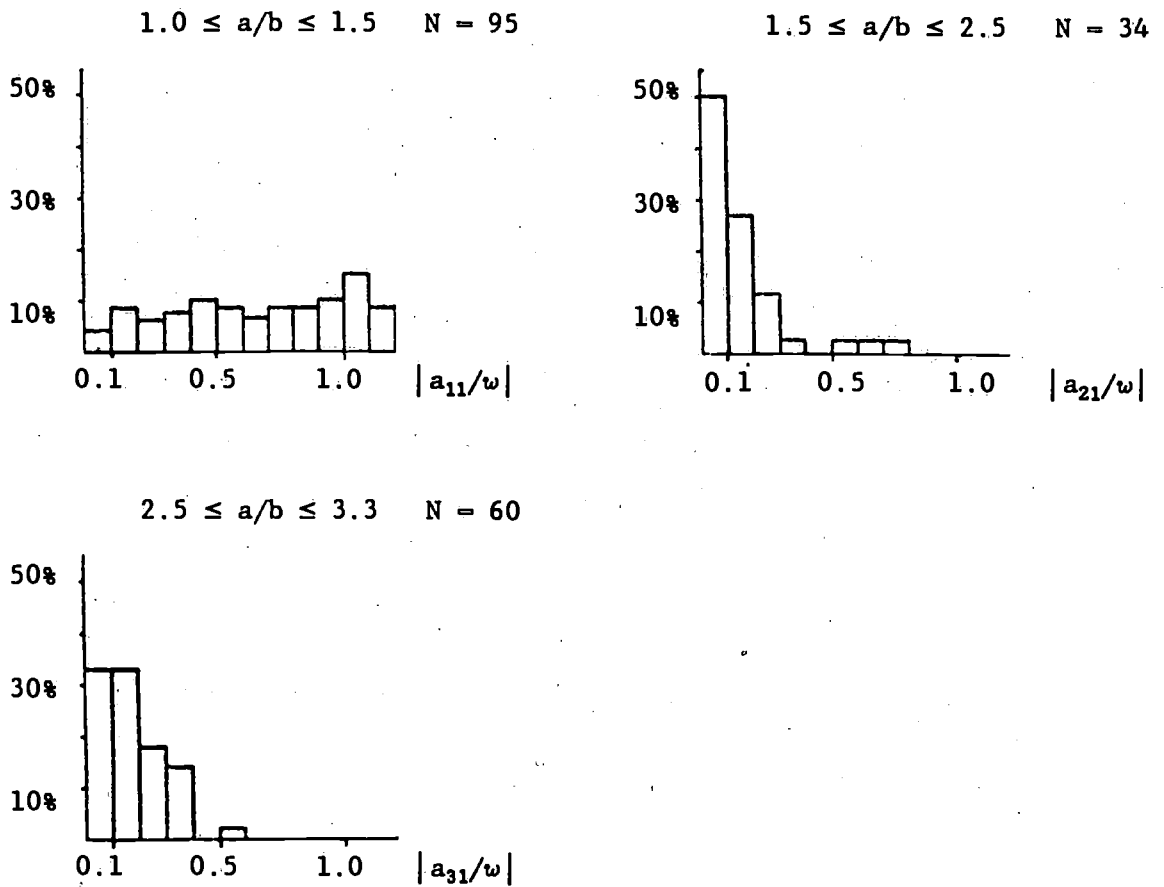


Fig. 14. Distributions of the amplitude of the buckling mode as a fraction of the maximum plate deflection for plates with a different aspect ratio.

TABLE 7

Distribution characteristics of harmful  $a_{n1}$  coefficients

harmful coefficient	mean value	standard deviation	measured minimum	measured maximum	aspect ratio
$ a_{11} /t$	0.20	0.14	0.02	0.53	1.00 + 1.41
$ a_{21} /t$	0.04	0.08	0.00	0.61	1.41 + 2.45
$ a_{31} /t$	0.05	0.04	0.01	0.23	2.45 + 3.46
$ a_{41} /t$	0.13	0.10	0.00	0.38	3.46 + 4.47
$ a_{11}/w $	0.70	0.29	-	-	1.00 + 1.41
$ a_{21}/w $	0.12	0.12	-	-	1.41 + 2.45
$ a_{31}/w $	0.15	0.13	-	-	2.45 + 3.46
$ a_{41}/w $	0.50	0.25	-	-	3.46 + 4.47

## 5. CONCLUSIONS

Initial plate deflections (IPD) affect the design, the production and the performance of a ship in different ways. It has been concluded that to properly foresee these effects, a statistical model consisting of two parts is necessary:

- model of the B-type maximum amplitude of IPD, for general use;
- model of the A-type maximum amplitude of IPD, for use when in-plane compressive strength has to be taken into account, because:
  - the measurement of the maximum plate deflection over a short gauge length, equal to, for instance, plate breadth  $b$ , at any point along the length of a plate provides a satisfactory representation of harmful wavelength distortions and localized dents with lengths in the range of  $0.5b-1.2b$ , without including the amplitude of less significant, longer wavelength distortions.
  - the concept of using the  $a_{h1}$  coefficients as a measure of the harmfulness of the real, complicated shapes of IPD, however although important from the point of view of the explanation of the effect of the shape of IPD on the compressive strength, has several important disadvantages.

A new model of the maximum B-type amplitude of IPD has been proposed. The model is obtained by careful statistical analysis of measured IPD.

The model defines:

- the mean value, the expected value and the commonly used cumulative values of the dimensionless maximum amplitude of IPD as the proportional function of the square of plate slenderness  $b/t$ . All these values can be obtained from one compact formula 2.7.2, depending on one tabulated factor  $k$  (Table 5);
- the log-normal distribution of the maximum amplitude of IPD for a given plate slenderness (relation 2.7.2);
- the 22% chance that the maximum amplitude of IPD is negative.

The validity of the model is ensured for plates with  $40 < b/t < 80$ . When  $b/t < 40$ , results of the model for  $b/t = 40$  may be used, and when  $b/t > 80$  the model may be extended up to  $b/t = 120$ .

There is acceptable agreement between the mean values of the maximum plate deflection defined by the new model and those defined by the majority of existing models.

~~One of the policies used to establish the tolerance limit for the acceptable maximum plate deflection is to take such a value that the probability of exceeding it is for example lower than 2.5%.~~

Following such a policy, the new tolerances for allowable maximum plate deflection are proposed (equations 3.2.1 and 3.2.3). The new tolerances, in comparison with existing tolerances, allow for considerably higher amplitudes of IPD. This difference is caused by the fact that previous investigators in defining the tolerance limit had been assuming the normal distribution of maximum plate deflection, which does not correspond to the real situation.

It is important that the new model describes the maximum amplitude of IPD which will occur in a structure if no special attempts have been made to prevent [33] or to reduce (fairing) them.

The model should be continuously re-examined for new data, in order to follow changes in welding technology and in the requirements for weld dimensioning.

The future work will include the analysis, the model and the proposal of new tolerances for the A-type amplitudes of IPD.

Presented here new model has been used in experimental and numerical part of the work to introduce representative amplitudes and shapes of initial deflections respectively into plate specimens and into the Finite Element models.

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